

Online Appendix

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This online appendix provides the proofs behind some of the results and claims in the main text.

A Proof of Proposition 2

Consider the case in Section 4.2 of the main paper. In the final stage, after θ is realized and the agent has set a price p , the agent chooses

$$q = \arg \max_{q_0} \left\{ (1-w) p D(p, q_0, Q) - \frac{1}{2} q_0^2 - t D(p, q_0, Q)^2 \right\}$$

and the principal simultaneously chooses

$$Q = \arg \max_{Q_0} \left\{ w p D(p, q, Q_0) - \frac{1}{2} Q_0^2 + t D(p, q, Q_0)^2 \right\}.$$

Provided $t \neq 0$ and second-order conditions hold, this leads to the equilibrium choices

$$\begin{aligned} q(p) &= \frac{\phi \left((1-w + 2t(\beta - \Phi^2)) p - 2t\theta \right)}{1 - 2t(\Phi^2 - \phi^2)} \\ Q(p) &= \frac{\Phi \left(2t\theta + (w - 2t(\beta - \phi^2)) p \right)}{1 - 2t(\Phi^2 - \phi^2)}. \end{aligned}$$

Evaluating the final stage choices of investments given p is set at the second-best level $p(\theta)$, and setting these equal to the second-best levels of investment we have that

$$q(p(\theta)) = \frac{\phi \left((1-w + 2t(\beta - \Phi^2)) p(\theta) - 2t\theta \right)}{1 - 2t(\Phi^2 - \phi^2)} = q(\theta)$$

and

$$Q(p(\theta)) = \frac{\Phi \left(2t\theta + (w - 2t(\beta - \phi^2)) p(\theta) \right)}{1 - 2t(\Phi^2 - \phi^2)} = Q(\theta)$$

provided

$$t(w) = \frac{(1-w)\Phi^2 - \phi^2 w}{2\beta(\phi^2 + \Phi^2) - 2\Phi^2\phi^2}.$$

Thus, provided t is set in this way (for any given w), at the second-best prices, the investments in the last stage will also both equal the second-best levels. Assumption (1) ensures the denominator is

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positive. Note in case $w < \frac{\Phi^2}{\Phi^2 + \phi^2}$, this implies $t > 0$, while if $w > \frac{\Phi^2}{\Phi^2 + \phi^2}$, this implies $t < 0$, which shows $w < 0$ and $t < 0$ is not possible.

Now consider the agent's pricing decision in the second stage. The agent solves

$$p = \arg \max_{p_0} \left\{ (1-w) p_0 D(p_0, q(p_0), Q(p_0)) - \frac{1}{2} q(p_0)^2 - t D(p_0, q(p_0), Q(p_0))^2 \right\}.$$

We need to make sure that w is set so that given $t(w)$ above, the optimizing choice of p_0 matches the second-best function $p(\theta)$.

Differentiating the agent's objective function with respect to p_0 we get

$$(1-w) \left(D + p_0 \frac{dD}{dp_0} \right) - q \frac{\partial q}{\partial p_0} - 2tD \frac{dD}{dp_0} = 0,$$

which can be evaluated at the second-best solutions $p_0 = p(\theta)$, $q = q(\theta)$ and $D = D(p(\theta), q(\theta), Q(\theta))$. Solving this first-order condition for w , after taking into account the relationship $t(w)$, implies

$$w = \frac{\sqrt{Y} - (2\Phi^4\phi^2 + \beta^2(\phi^2 + \Phi^2) - 2\Phi^2\beta(\phi^2 + \Phi^2))}{2(\Phi^2 - \phi^2)(\beta(\phi^2 + \Phi^2) - \phi^2\Phi^2)}$$

where

$$Y = \beta^4(\Phi^2 + \phi^2)^2 + 4\Phi^4\phi^4(4\beta^2 + \Phi^2\phi^2) + 4\beta\phi^2\Phi^2(\beta(\phi^4 + \Phi^4 - 2\beta(\phi^2 + \Phi^2)) - 2\phi^2\Phi^2(\phi^2 + \Phi^2)).$$

At this level of w and $t(w)$, the agent will chose p_0 to be equal to the second-best level $p(\theta)$, and the final stage investments q and Q will also replicate the second-best solution. Note using L'Hôpital's rule, when $\phi^2 \rightarrow \Phi^2$ we can confirm that $w = \frac{1}{2}$ and $t = 0$, as expected. Numerical examples show that even under (1), w can be negative or t can be negative.

B Proof of Proposition 3

In the final stage we have

$$\begin{aligned} q &= \arg \max_{q_0} \left\{ (1-w) p(\theta - \beta p + \phi q_0 + \Phi Q) - \frac{1}{2} q_0^2 \right\} = \phi(1-w)p \\ Q &= \arg \max_{Q_0} \left\{ w p(\theta - \beta p + \phi q + \Phi Q_0) - \frac{1}{2} Q_0^2 \right\} = \Phi w p. \end{aligned}$$

Then in the second stage, the agent chooses

$$p(w, \theta) = \arg \max_{p_0} \left\{ (1-w) p_0(\theta - \beta p_0 + \phi q(p_0) + \Phi Q(p_0)) - \frac{1}{2} q(p_0)^2 \right\},$$

which implies

$$p(w, \theta) = \frac{\theta}{2\beta - \phi^2 + (\phi^2 - 2\Phi^2)w}.$$

The second-order condition is $2\beta - \phi^2 + (\phi^2 - 2\Phi^2)w > 0$ and we will verify later that it holds at the optimal value of w determined below. The agent's ex-post payoff for any given θ is

$$\frac{(1-w)\theta^2}{2(2\beta - \phi^2 + w(\phi^2 - 2\Phi^2))} > 0.$$

Thus, the principal's first stage optimization problem is

$$\begin{aligned} & \max_{w,T} \left\{ \mathbb{E}_\theta \left[wp(w, \theta) (\theta - \beta p(w, \theta) + \phi q(p(w, \theta)) + \Phi Q(p(w, \theta))) - \frac{1}{2} Q(p(w, \theta))^2 \right] + T \right\} \\ & \text{subject to} \\ T & \leq \mathbb{E}_\theta \left[(1-w)p(w, \theta) (\theta - \beta p(w, \theta) + \phi q(p(w, \theta)) + \Phi Q(p(w, \theta))) - \frac{1}{2} q(p(w, \theta))^2 \right], \end{aligned}$$

which reduces to

$$\begin{aligned} & \max_w \left\{ \mathbb{E}_\theta \left[p(w, \theta) (\theta - \beta p(w, \theta) + \phi q(p(w, \theta)) + \Phi Q(p(w, \theta))) - \frac{1}{2} q(p(w, \theta))^2 - \frac{1}{2} Q(p(w, \theta))^2 \right] \right\} \\ = & (\theta^2 + V_\theta) \max_w \left\{ \frac{(2\beta - w^2(\Phi^2 + \phi^2) - 2w(\Phi^2 - \phi^2) - \phi^2)}{2(2\beta - \phi^2 - 2w\Phi^2 + w\phi^2)^2} \right\}. \end{aligned}$$

The derivative in w is

$$(\theta^2 + V_\theta) \frac{(2\beta - \phi^2)\Phi^2 - w(2\beta(\Phi^2 + \phi^2) + 2\Phi^4 - 4\Phi^2\phi^2)}{(2\beta - \phi^2 - 2w\Phi^2 + w\phi^2)^3}$$

and is equal to zero for

$$w^* = \frac{\Phi^2(2\beta - \phi^2)}{2\beta(\phi^2 + \Phi^2) + 2\Phi^4 - 4\Phi^2\phi^2}.$$

We need to verify three things to ensure that this w^* does indeed maximize the principal's profits: i) $0 < w^* < 1$, ii) the second-order condition in w for the principal's optimization problem, and iii) the second-order condition in p_0 for the agent's optimization problem.

First, note that (1) implies $2\beta > \phi^2$ and the first inequality in

$$2\beta(\phi^2 + \Phi^2) > \Phi^4 + 2\Phi^2\phi^2 + \phi^4 > 4\Phi^2\phi^2 - 2\Phi^4.$$

Thus, $w^* > 0$. Furthermore, $w^* < 1$ is equivalent to

$$2\beta\phi^2 > 3\Phi^2\phi^2 - 2\Phi^4.$$

Assumption (1) implies the first inequality of

$$2\beta\phi^2 > \frac{(\phi^4 + \Phi^4 + \phi^2\Phi^2)\phi^2}{\phi^2 + \Phi^2} > 3\Phi^2\phi^2 - 2\Phi^4,$$

where the second inequality is true for all non-negative ϕ and Φ . Thus, we have proven that $0 < w^* < 1$

under assumption (1).

Second, we have

$$2\beta - \phi^2 - 2w^*\Phi^2 + w^*\phi^2 = \frac{(2\beta - \phi^2)(2\beta(\Phi^2 + \phi^2) - 3\Phi^2\phi^2)}{2\beta(\phi^2 + \Phi^2) + 2\Phi^4 - 4\Phi^2\phi^2} > 0,$$

where $2\beta - \phi^2 > 0$, $2\beta(\Phi^2 + \phi^2) - 3\Phi^2\phi^2 > 0$ and $2\beta(\phi^2 + \Phi^2) + 2\Phi^4 - 4\Phi^2\phi^2 > 0$ all follow from (1). Thus, the second-order condition of the principal's optimization problem in w holds when evaluated at $w = w^*$.

Third and finally, the second-order condition of the agent's optimization problem in p evaluated at $p_0 = p(w^*, \theta)$ holds for all θ if and only if

$$2\beta - \phi^2 + (\phi^2 - 2\Phi^2)w^* > 0.$$

This is equivalent to

$$\frac{(2\beta - \phi^2)(2\beta(\Phi^2 + \phi^2) - 3\Phi^2\phi^2)}{2\beta(\phi^2 + \Phi^2) + 2\Phi^4 - 4\Phi^2\phi^2} > 0,$$

which is also implied by assumption (1).

Evaluating the principal's expected profits at w^* we obtain

$$\mathbb{E}_\theta [\Pi(\theta)] = \frac{(2\beta(\phi^2 + \Phi^2) + \Phi^4 - 3\phi^2\Phi^2)(\bar{\theta}^2 + V_\theta)}{2(2\beta - \phi^2)(2\beta(\phi^2 + \Phi^2) - 3\Phi^2\phi^2)}.$$

Comparing to the second-best profit given by (5) in the main paper, the difference in expected profit divided by the second-best profit is

$$\frac{\Phi^4(\Phi^2 - \phi^2)^2}{(2\beta - \phi^2)(2\beta(\phi^2 + \Phi^2) - 3\phi^2\Phi^2)(\phi^2 + \Phi^2)}.$$

This represents the principal's percentage loss relative to second-best. It is then easily verified that this expression is equal to the one provided in the text of Proposition 3.

C Alternative timing and misreporting of price

Consider the alternative specification in Section 5.3 of the main paper. Consider a contract that involves the per-unit wholesale price being equal to $w p_r$, based on the reported price, and a payment $f p_r^2$ that only depends on the reported price in stage 2 and not the number of units sold, as well as a penalty $f_p(p^2 - p_r^2)$. The penalty only applies if the price is actually observed, which is assumed to happen with probability η . The principal determines the contract parameters w , f , and f_p , while η is an exogenous parameter.

The equilibrium in stage 3 is defined by

$$\begin{aligned}
p(\theta, w, f_p, p_r) &= \arg \max_p \{ (p - wp_r) (\theta - \beta p + \phi q(\theta, w, f_p, p_r) + \Phi Q(w, p_r)) - \eta f_p (p^2 - p_r^2) \} \\
&= \frac{\theta + \phi q(\theta, w, f_p, p_r) + \Phi Q(w, p_r) + \beta wp_r}{2(\beta + \eta f_p)} \\
q(\theta, w, f_p, p_r) &= \phi (p(\theta, w, f_p, p_r) - wp_r) \\
Q(w, p_r) &= \Phi wp_r.
\end{aligned}$$

The stage 3 equilibrium is then

$$\begin{aligned}
p(\theta, w, f_p, p_r) &= \frac{\theta + wp_r (\beta + \Phi^2 - \phi^2)}{2(\beta + \eta f_p) - \phi^2} \\
q(\theta, w, f_p, p_r) &= \frac{\theta \phi - wp_r \phi (\beta + 2\eta f_p - \Phi^2)}{2(\beta + \eta f_p) - \phi^2} \\
Q(w, p_r) &= \Phi wp_r.
\end{aligned} \tag{C.1}$$

Assuming that f is chosen to induce truthful revelation of p such that $p = p_r$ in equilibrium (which we'll show later is possible), the stage 3 equilibrium as a function of θ , w and f_p is

$$\begin{aligned}
p &= p_r = \frac{\theta}{2(\beta + \eta f_p) - \phi^2 - w(\beta + \Phi^2 - \phi^2)} \\
q &= \frac{\theta \phi (1 - w)}{2(\beta + \eta f_p) - \phi^2 - w(\beta + \Phi^2 - \phi^2)} \\
Q &= \frac{\theta \Phi w}{2(\beta + \eta f_p) - \phi^2 - w(\beta + \Phi^2 - \phi^2)}.
\end{aligned}$$

Recalling that the second-best solution is fully characterized by (3)-(4), we can match the second-best solution by setting

$$\begin{aligned}
w &= \frac{\Phi^2}{\phi^2 + \Phi^2} \\
f_p &= \frac{\Phi^2 (\beta - \phi^2)}{2\eta (\phi^2 + \Phi^2)}.
\end{aligned}$$

Note that at this value of f_p , the second-order condition necessary for the optimizations above to be valid holds if and only if

$$2\beta + 2\eta f_p - \phi^2 = \frac{(2\beta - \phi^2) (\phi^2 + \Phi^2) - \Phi^4 + \Phi^2 (\beta + \Phi^2 - \phi^2)}{\phi^2 + \Phi^2} > 0.$$

Finally, we need to ensure truthful revelation of the price by the agent in stage 2. The agent's

profit from the perspective of stage 2 is

$$(p(\theta, w, f_p, p_r) - wp_r)(\theta - \beta p(\theta, w, f_p, p_r) + \phi q(\theta, w, f_p, p_r) + \Phi Q(w, p_r)) - \frac{1}{2}q(\theta, w, f_p, p_r)^2 - \eta f_p p(\theta, w, f_p, p_r)^2 - (f - \eta f_p) p_r^2.$$

Using (C.1), the level of p_r that maximizes these profits is

$$p_r(\theta, w, f_p, f) = \frac{\theta w (2\eta f_p + \beta - \Phi^2)}{w^2 \left((\beta - \Phi^2)^2 - 2\eta f_p (2\Phi^2 - \phi^2) \right) - 2(f - \eta f_p) (2\beta + 2\eta f_p - \phi^2)}.$$

To induce truthful revelation, we need to have

$$p(\theta, w, f_p, p_r) = p_r(\theta, w, f_p, f)$$

for all θ when $w = \frac{\Phi^2}{\phi^2 + \Phi^2}$ and $f_p = \frac{\Phi^2(\beta - \phi^2)}{2\eta(\phi^2 + \Phi^2)}$. This can be achieved by setting

$$f = \frac{\phi^2 \Phi^2 (\Phi^2 - \phi^2)}{2(\Phi^2 + \phi^2)^2},$$

which is identical to the level of f in (8), which corresponds to the optimal ad valorem contract in the benchmark setting. Taking into account that $p_r > 0$ and $w > 0$ at the proposed equilibrium, the second-order condition corresponding to the last optimization problem (in p_r) is

$$\beta - \Phi^2 + 2\eta f_p = \frac{\beta (2\Phi^2 + \phi^2) - \Phi^2 (\Phi^2 + 2\phi^2)}{\phi^2 + \Phi^2} < 0.$$

Thus, the solution we have determined above is valid provided

$$\begin{aligned} (2\beta - \phi^2) (\phi^2 + \Phi^2) - \Phi^4 + \Phi^2 (\beta + \Phi^2 - \phi^2) &> 0 \\ \Phi^2 (\Phi^2 + 2\phi^2) - \beta (2\Phi^2 + \phi^2) &> 0. \end{aligned}$$

Since there is no penalty in equilibrium (recall $p = p_r$ with this contract), and since p , q and Q are all set at the second-best levels and the upfront fee T is set according to (9) so as to extract the agent's expected profit, the principal will achieve expected profits as defined in (5), consistent with the second-best outcome.