Data-enabled learning, network effects and competitive advantage*

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Abstract

We model dynamic competition between firms which improve their products through learning from customer data, either by pooling different customers' data (across-user learning) or by learning from repeated usage of the same customers (within-user learning). We show how a firm's competitive advantage is affected by the shape of firms' learning functions, asymmetries between their learning functions, the extent of data accumulation, and customer beliefs. We also explore how public policies towards data sharing, user privacy and killer data acquisitions affect competitive dynamics and efficiency. Finally, we show conditions under which a consumer coordination problem arises endogenously from data-enabled learning.

Keywords: machine learning, dynamic competition, data sharing, switching costs.

1 Introduction

In recent years, much attention has been focused on the role data can play in providing incumbent firms with a competitive advantage. Digitization, connectivity to cloud-based infrastructures, together with cheaper storage and more effective use of data (i.e. improvements in machine learning algorithms), have made it possible for firms in many industries to translate learning from their customer data into rapid improvements in their products on

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a scale that has not been seen before. And with better products, these firms can attract more customers (or more usage from existing customers), and therefore obtain more data, potentially creating a self-reinforcing cycle that can make it difficult for any new entrant to compete. We call this self-reinforcing cycle "data-enabled learning".[1]

There are a plethora of products and services that leverage data-enabled learning. Some work purely by leveraging data across different users, whereas others personalize the learning at the individual level. As an example of the former, consider Grammarly. This well-known cloud-based service suggests spelling, grammar, tone, style and word choice improvements in exchange for a monthly subscription fee. As users make corrections to the suggestions offered by Grammarly, its language experts and artificial intelligence can use this feedback to continue to improve its future recommendations for all users. Other text processing applications such as chatbots (LivePerson), document management (Luminance) and productivity tools (x.ai) work in a similar way, as do computer vision applications such as driver assistance (Mobileye), autonomous vehicle systems (Cruise), skin cancer detection (SkinVision), farm monitoring (Prospera), home security (Deep Sentinel) and airport screening (SeeTrue), and speech recognition applications such as dictation software (Dragon), translation services (Unbabel) and virtual assistants (Amelia).

In contrast to these applications based on across-user learning, consider Fitbit's premium service, which provides users with personalized advice on health, sleep and fitness based on data collected by their Fitbit device. As customers use their Fitbit device more, the system is able to provide them with better and more helpful advice, which means users tend to prefer it to any new alternative device. And of course, many applications work by combining both across-user learning and within-user learning, such as paid services that are built on recommendation systems like those offered by Calm, Netflix, Spotify, StitchFix and Tinder.

Data-enabled learning would seem to give incumbent firms a competitive advantage. But how strong is this advantage and how does it differ from that obtained from more traditional mechanisms, such as (i) learning-by-doing, which allows a firm to lower its costs as it produces (and sells) more, thereby being able to offer a lower price and attract more sales, and (ii) standard network effects, which make a firm more valuable to consumers as it attracts more consumers, thereby attracting even more consumers and offering even higher value?

To address these questions, and determine the drivers of competitive advantage with data-enabled learning, we develop a theory of dynamic competition in which firms can improve their products via data gleaned from their customers. The theory is based on Bertrand

[1] Some practitioners have called this phenomenon "data network effects". Our use of "data-enabled learning" is intended to be more general because, as discussed below, the phenomenon need not involve any network effect (for example, if the learning is specific to each customer).

competition between two infinitely lived firms, which can differ in the amount of their prior learning as well as in the shape of their respective learning functions. We allow firms to set prices to compete for consumers every period, taking into account how attracting consumers (and therefore more data) increases the value they can offer to consumers in subsequent periods. An important feature of our framework is its ability to handle asymmetric learning functions. This generality is achieved by simplifying the model in another dimension — there is no uncertainty determining which firm has an intrinsic advantage in every given period, which means the market always has a winner-takes-all property, with the same firm winning in every period.

As a baseline, we consider the case with across-user learning only (Section 3), in which the two firms only learn based on the total number of consumers they served in the past. Because consumers' willingness-to-pay increases over time with more users when prices remain unchanged, there is a type of dynamic network effect at play. This provides one distinction from traditional learning-by-doing settings, where consumers benefit only via lower prices. A more substantive distinction arises when we allow firms to learn and improve their products while their customers are still consuming them. This is often the case with cloud-based products, but was not possible with traditional products (e.g. physical goods, packaged software). When the product improves while it is still being consumed, consumers need to form expectations over how many other consumers will purchase the same product in order to determine the value they will get from the product, thereby creating a role for consumer beliefs.

We then consider the case of within-user learning only (Section 4), in which any given consumer's willingness-to-pay for a given firm's product only depends on how many times the firm has served that particular consumer in the past. This highlights another fundamental difference between data-enabled learning and traditional learning-by-doing mechanisms, which is the role of customization: firms can often improve their products for each individual customer based on that customer's particular usage experience. Within-user learning implies the more a customer uses a firm's product, the greater is the effective cost of switching to the rival's product. Such data-enabled learning therefore creates a switching cost that grows over time, but not any type of network effect.

In these baseline models, we show that despite the fact that sometimes both the winning firm and the losing firm price below cost to consumers, the competitive outcome coincides with the socially optimal outcome in terms of which firm wins in each period.[2] The condition determining which firm will win is the same regardless of whether we assume pure across-

[2]The only exception to this occurs with across-user learning when products can improve while they are still being consumed, so there is a role for consumer beliefs to distort the equilibrium outcomes.

user learning or pure within-user learning. However, the profits of the winning firm are different: namely, the winning firm makes strictly higher profits when learning is purely across users relative to the case when it is purely within users. Fundamentally, this is because within-user learning creates an increasing switching costs, so each forward-looking individual consumer takes into account that they would benefit if the losing firm had a stronger competitive position in the future, which in turn makes the winning firm have to price more competitively. We also explain how various factors, such as how far each firm is along its learning curve and the nature of the learning curve, affect the condition for either firm to win.

In Section 5 we show how combining the two types of learning (across users and within users) can make consumers' beliefs matter for the equilibrium outcome. With myopic consumers we show things remain equivalent to the case of pure across-user learning, in which consumer beliefs don't matter. However, once we allow for forward-looking consumers, beliefs can matter provided the value of across-user learning is sufficiently large relative to the value of within-user learning.

A key implication of our analysis is that providing the losing firm with more data generally increases consumer surplus. This might suggest a policy that forces winning firms to share the data obtained with their rivals would be good for consumers. We use our dynamic framework to explore how data sharing works: we find that it increases consumer surplus when one firm is sufficiently far ahead of the other by making the laggard more competitive, but it decreases consumer surplus when the firms are sufficiently evenly matched by making firms compete less aggressively, which in our model means subsidizing consumers less. We also use our model to highlight an unintended consequence of privacy policies. If such policies reduce the rate at which firms can extract useful data from consumers, they will tend to increase the incumbent's competitive advantage, reflecting that the entrant has more scope for new learning and so is affected more by such a policy. Finally, our baseline efficiency result no longer necessarily holds when the firms can acquire data directly (e.g. from a third party) rather than via attracting more consumers. Indeed, we show that if one firm has already reached its maximum learning threshold, that firm will acquire the available data too often relative to what is socially efficient. Thus, the theory predicts killer data acquisitions, in which the incumbent will acquire data even though it has no use for it, because it stands to gain more by denying the rival access to the data than the rival would obtain by catching up.

2 Related literature

Our article combines aspects from three large and established literatures. First, it relates to works that provide theoretical models of competition with learning-by-doing (Fudenberg and Tirole, 1983; Dasgupta and Stiglitz, 1988; Salant, 1990; Cabral and Riordan, 1994; Besanko et al., 2010; and Besanko et al., 2014 and 2019). Our treatment of pure across-user learning (in the case without within-period learning), in which learning increases consumers' willingness to pay in subsequent periods, can be viewed as isomorphic to the decreases in firms' costs arising under traditional learning-by-doing models. In this sense, the baseline model with pure across-user learning is close to that in the classic article of Cabral and Riordan (1994), and the articles by Besanko et al., 2010, and Besanko et al., 2014 and 2019 that build on their framework. Relative to these articles, our framework is simpler by assuming away uncertainty (i.e. firm-specific shocks to consumer valuations). Without a stochastic element determining which firm has an intrinsic advantage in every period, our model is less useful in terms of predicting the evolution of market shares in an industry. On the other hand, we contribute to the learning-by-doing literature with our focus on asymmetric learning functions. This creates the possibility that a firm that currently has a smaller stock of data but access to a superior learning technology can have a competitive advantage that enables it to win current and future sales.

A benefit of our model of learning is we can explicitly solve for prices and value functions in every state for arbitrary learning functions, which is what allows us to obtain a wide range of new results, including the implications of learning for consumer surplus, and comparative statics in the shape of the firms' respective learning curves and in their respective positions on those curves. And more fundamentally, we apply the framework to study questions that do not naturally arise in learning-by-doing settings, specifically policy questions regarding the implications of data sharing, tighter customer privacy, and data acquisitions. Furthermore, the within-user learning version of our model is entirely new, as is the role of switching costs in that setting. Finally, a further novelty of data-enabled learning relative to learning-by-doing is the possibility for consumer coordination problems to arise, such as when we combine across-user learning with within-period learning or within-user learning.

Second, our article relates to the substantial literature on network effects. From this literature we borrow the modelling of consumer "beliefs" to select equilibria when consumers play coordination games (Katz and Shapiro, 1986, and Caillaud and Jullien, 2001). At a high level, our model with across-user and within-period learning is related to dynamic models with network effects (e.g. Mitchell and Skrzypacz, 2006, Cabral, 2011, Biglaiser and Cremer 2020, Halaburda et al., 2020), as the combination of these two features works in the same way as exogenous network effects within a period. However, a key difference is that in our setting with learning, willingness-to-pay depends not only on the current stock of consumers, as it would in network effects models, but also on past consumers even if they are no longer consuming from the firm.

Third, our article relates to IO models of switching costs, such as the classic articles of Klemperer (1987) and Beggs and Klemperer (1992). As noted in the introduction, within-user learning creates a switching cost that grows the longer a consumer is with a particular firm. This contrasts with the standard assumption in the literature that switching costs are constant. Nilssen (1992) allows that switching costs are higher when switching to a new firm as opposed to a firm that has been previously tried, but doesn't allow switching costs to continue to increase in the number of periods a consumer has purchased from a given firm. To the best of our knowledge, this feature has not previously been considered in the switching cost literature even though it seems relevant more generally.

Finally, there is a rapidly emerging literature on the role and implications of data in the economy. One branch of this literature (see Acemoglu et al., 2022, Bergemann et al., 2022, Choi et al., 2019, and Ichihashi, 2020 and 2021) studies the negative implications of firms collecting consumer data when that data can be used in ways that harm consumer privacy (e.g. selling it to third parties), thus giving rise to the possibility of excessive data collection. In contrast, and consistent with our motivating examples, the firms collecting data from their customers in our model use that data only to improve their own products for consumers. Articles that also focus on data-enabled learning include Farboodi et al. (2019), who model the industry dynamics that arises when data helps firms choose the best production technique and so higher quality products, and Prufer and Schottmüller (2022), who model a dynamic feedback loop in which a firm's current cost of investing in quality is decreasing in the firm's previous period sales, and greater investment in quality leads to higher demand in the current period. Both abstract from strategic price setting by firms and explicit modelling of the decisions of forward looking consumers. Biglaiser et al. (2019) provides a high-level discussion of the different ways firms can enjoy incumbency advantages, including access to more data: they note that across-user learning and within-user learning are two distinct ways in which data generates a competitive advantage for incumbents. De Cornière and Taylor (2021) also study the effect of data on competition but focus on showing how different uses of data (including improving a firm's product, but also ad targeting and price discrimination) determine whether data is pro- or anti-competitive. Although their framework is much more general in other ways, they don't analyze a fully dynamic setting in which data learning can accumulate. Finally, Schaefer and Sapi (2022) investigate how data drives the quality of internet search results, providing evidence that the quality of search results improve with more data on previous searches, and that personalized information is particularly valuable.

3 Across-user learning

In this section we focus on across-user learning—each firm improves the product for each consumer based on what it learns from the usage of all its consumers. The learning technology we consider has the general property that the value to consumers of a given firm's product in a period is an increasing function of the measure of consumers who have purchased the product previously (past consumers) from the same firm. We will refer to the model described below as our baseline setting because it turns out that much of the model setup and analysis, as well as many of the results in this section, carry over (with suitable adjustments) to the versions where we allow for within-period learning (at the end of this section), and within-user learning, either by itself (Section 4), or in combination with across-user learning (Section 5).

Two firms, which we refer to as the incumbent I and the entrant E, compete over infinitely many periods. Both firms face a marginal cost of production equal to c and compete in prices. We assume consumers have unit demands in each period, which they can fulfill either from I or E. Although firms charge positive prices in all of our motivating examples, we can allow for the possibility of negative prices by reinterpreting the price as some valuable service that the firm offers consumers in exchange for their data and which is costly for the firm to provide. The (common) one period ahead discount factor is denoted δ, and satisfies $0 < \delta < 1$. We normalize the measure of consumers in each period to one, and so the number of periods that a firm has previously won also represents the total measure of consumers it can learn from. In this formulation of across-user learning, there is no distinction between consumers who are purchasing again (repeat consumers) vs. entirely new consumers. Thus, our setting in this section is compatible with two interpretations:

- Consumers live for one period only. New consumers in a period make a single buying decision and then exit.
- The same consumers are active and make purchasing decisions in every period, i.e. they are infinitely lived and forward looking, but there is no cost for each of these individual consumers to switch from one firm to the other across periods.

An important implication of consumers being atomistic is that even if consumers are of the second type above, they do not need to consider the impact of their purchase decision on the pricing game or the options they will face in subsequent periods. Thus, consumers' optimal choices can be made by comparing the surplus offered by each firm in the current period only, which is why the same results would arise if we had assumed long-lived consumers were myopic instead. Later, when we introduce within-user learning, this will no longer be true.

Consumers are identical in their preferences for the two firms.[3] Specifically, the standalone value of firm i's product (or service) is denoted s_i, where $i \in \{I, E\}$. This value is augmented by learning. In any period, firms are asymmetric Bertrand competitors, so each firm will either sell to all consumers (measure one) or none. If firm i has sold in N_i periods in the past, then the value current consumers obtain from firm i's product is $s_i + f_i(N_i)$, where f_i is firm i's learning function. We assume that firm i reaches its maximum threshold of learning after selling in $\overline{N}_i$ periods, i.e. there exists $\overline{N}_i \geq 1$ such that $f_i(N_i) = f_i\left(\overline{N}_i\right)$ for all $N_i \geq \overline{N}_i$.

This specification captures that firms learn more as they obtain more consumer data, and this allows them to increase the value of the product they offer, but only up to a point, i.e. learning is bounded. Later in this section we will show how our results extend to handle unbounded learning functions. The only requirements on f_i are that it is defined for all non-negative real values of its argument, is everywhere weakly increasing, and without loss of generality, strictly increasing in the last step to reach its maximum threshold (i.e. $f_i(\overline{N}_i) > f_i(\overline{N}_i - 1)$). In particular, f_i need not be a continuous function. We normalize $f_i(0) = 0$. Thus, at any point in time there are potentially three sources of asymmetry between firms: (i) differences in standalone value ($s_I \neq s_E$), (ii) differences in the learning curve ($f_I \neq f_E$ and/or $\overline{N}_I \neq \overline{N}_E$), and (iii) differences in how far along each firm is on its learning curve in the current period ($N_I \neq N_E$).[4]

Consumers and firms have perfect and complete information. Our focus is on Markovian strategies and a Markov-perfect equilibrium (MPE). Among MPEs, we rule out equilibria which are supported by the losing firm pricing in such a way that its value would be negative if it instead won consumers at the prices charged. When we refer to a unique MPE we mean the MPE is unique after eliminating MPEs that involve such weakly dominated strategies. Finally, we use the convention that E wins in case consumers are indifferent when both firms price at the lowest level they would still be willing to sell at.

[3] This is distinct from the data each consumer provides to firms, which could still be different across consumers.

[4] In Cabral and Riordan (1994), there are no differences in learning curves across firms, but rather, they allow for horizontal differentiation via firm-specific shocks to consumer valuations. If we introduced the above asymmetries in standalone values and learning curves into their framework, and take the limit as the variance of their stochastic term goes to zero, we would obtain a model that is isomorphic to ours.

An implicit assumption in our setup is that a firm will continue to compete even if in equilibrium its value is zero. This is consistent with our assumption that firms do not face any fixed cost to remain in operation and ensures firms always face some form of competition. There are many realistic ways to extend our model such that the losing firm makes positive expected profits in every period and therefore remains active provided fixed operating costs are not too large. In Online Appendix A, we present the details for two such extensions, as well as explaining what happens when the losing firm exits.

The relevant state at the start of any period is defined by (N_I, N_E). The corresponding value functions for I and E (i.e. the present discounted values (PDVs) of future profit flows starting with the current period when firms follow their equilibrium pricing strategies in every period) are denoted by $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$. Consider firms starting from the state $(\overline{N}_I, \overline{N}_E)$, so both firms have reached their respective learning thresholds.[5] Then I offers $s_I + f_I(\overline{N}_I)$ to consumers in every period and E offers $s_E + f_E(\overline{N}_E)$ to consumers in every period. Given the resulting asymmetric Bertrand competition, and taking into account that the competitive situation will be the same in every subsequent period, E wins every period starting with the current one if and only if $s_E + f_E(\overline{N}_E) \geq s_I + f_I(\overline{N}_I)$. Furthermore, for each firm $i \in \{I, E\}$ and its rival $j \in \{I, E\}$ with $j \neq i$, we must have

$$V^i(\overline{N}_I, \overline{N}_E) = \frac{1}{1-\delta} \max\left\{s_i - s_j + f_i(\overline{N}_i) - f_j(\overline{N}_j), 0\right\}. \tag{1}$$

When at least one of the firms has not yet reached its learning threshold, in order to win the current period, each firm will be willing to offer a subsidy equal to the difference in the discounted value of its future profits if it were to win the current period and its future profits if it were to lose the current period. Taking into account the maximum subsidy each firm is willing to offer in order to win, we can work out which firm will win in each period and its corresponding value function, working backwards from $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$. Using this approach, we show that for any state (N_I, N_E), there is a cutoff level of $s_E - s_I$ above which E wins and below which I wins. We use two-dimensional backwards induction in (N_I, N_E) to characterize this cutoff level (unless stated otherwise, all proofs are in the Appendix).[6]

Proposition 1. *Suppose I has previously sold to $N_I \geq 0$ consumers and E has previously sold to $N_E \geq 0$ consumers. There exists a unique MPE in which E wins in all periods if and*

[5] Due to the learning thresholds of the two firms, any state (N_I, N_E) with $N_I \geq \overline{N}_I$ is equivalent to the state $(\overline{N}_I, N_E)$ and any state (N_I, N_E) with $N_E \geq \overline{N}_E$ is equivalent to the state $(N_I, \overline{N}_E)$.

[6] The firms' corresponding value functions are given in the proof of Proposition 1.

only if $s_E - s_I \geq \Delta(N_I, N_E)$, *and I wins in all periods otherwise, where*

$$\Delta(N_I, N_E) = (1-\delta)\left(\sum_{j=0}^{\infty}\delta^j f_I(N_I+j) - \sum_{j=0}^{\infty}\delta^j f_E(N_E+j)\right). \tag{2}$$

This equilibrium continues to exist in the limit as $\overline{N}_I \to \infty$ *and* $\overline{N}_E \to \infty$ *(i.e. such that the learning functions continue to increase forever) provided there exists some power function which the learning curves lie below in the limit.*

Proposition 1 characterizes I's competitive advantage, which is captured by $\Delta(N_I, N_E)$.[7] The higher is $\Delta(N_I, N_E)$, the higher is the level of $s_E - s_I$ required for E to win. After dividing through by $1-\delta$, the expression $\Delta(N_I, N_E)$ is equal to the difference in the PDV of gross surplus generated from learning across the two firms when comparing the paths where each firm wins in every period from the current period onwards.

Clearly $\Delta(N_I, N_E)$ is increasing in N_I and decreasing in N_E. This captures that along the equilibrium path, if a firm wins in the current period, because it benefits from an additional learning period, $\Delta(N_I, N_E)$ shifts such that the same firm will continue to win in each subsequent period, and indeed does so even more easily. This last feature is reminiscent of the "increasing increasing dominance" property explored by Cabral and Riordan (1994). Yet in our setting, E may win and so move ahead of I even though it is currently behind (e.g. $N_E < N_I$). This could be because it learns more despite having less data (so $f_E(N_E) > f_I(N_I)$), or even if $f_E(N_E) < f_I(N_I)$, this could be because its learning curve is steeper and/or is increasing for longer, and so ultimately generates more value from getting ahead. For example, inspecting (2) and recalling that $f_i(N_i) = f_i(\overline{N}_i)$ for all $N_i \geq \overline{N}_i$, one can infer that if $f_E(\overline{N}_E) > f_I(\overline{N}_I)$, then for δ sufficiently close to 1 we have $\Delta(N_I, N_E) < 0$, so no matter how far behind it is today, E has an overall competitive advantage.

As noted in Proposition 1, the equilibrium we have characterized still exists in the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$. This allows us to capture any unbounded learning function provided it does not increase faster than some power function.[8] Thus, for instance, if the learning functions are linear without bounds (i.e. $f_i(N_i) = \theta_i N_i$), Proposition 1 implies

$$\Delta(N_I, N_E) = \theta_I\left(N_I + \frac{\delta}{1-\delta}\right) - \theta_E\left(N_E + \frac{\delta}{1-\delta}\right).$$

Our framework allows us to derive comparative statics taking into account asymmetries

[7] Note that $\Delta(N_I, N_E) = \Delta(\overline{N}_I, N_E)$ for all $N_I \geq \overline{N}_I$ and $\Delta(N_I, N_E) = \Delta(N_I, \overline{N}_E)$ for all $N_E \geq \overline{N}_E$.

[8] By the same logic underlying this result, all our characterizations of $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ in this article also extend to this unbounded case, by taking the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$.

across the firms' learning functions. From (2), I's competitive advantage is increasing in the maximum threshold for I's learning $\overline{N}_I$, decreasing in the maximum threshold for E's learning $\overline{N}_E$, increasing in anything that increases I's learning function f_I (e.g. an improvement in I's learning technology) and decreasing in anything that increases E's learning function f_E. Furthermore, a common technological improvement that improves both firms' learning functions by the same multiplicative factor $\theta > 1$ enhances the competitive advantage of the firm that has an advantage to begin with (i.e. I if $\Delta(N_I, N_E) > 0$ and E if $\Delta(N_I, N_E) < 0$). Thus, for instance, if both firms have access to the same learning technology, and I has more data to start with ($N_I > N_E$), its competitive advantage will increase if their common learning function increases by the same multiplicative factor $\theta > 1$. This reflects that data and a multiplicative technological improvement are inherently complementary. This is also why if I has a proportionally better learning technology (i.e. $f_I(N_k) = \theta f_E(N_k)$ for any N_k, with $\theta > 1$) and both firms start with the same amount of data ($N_I = N_E = N$), then increasing that initial amount of data increases I's competitive advantage. We derive additional comparative static results by putting more structure on the learning functions in the next subsection.

An interesting implication of the proof of Proposition 1 is that the losing firm may be still willing to offer a subsidy to consumers provided the winning firm's competitive advantage is not too large. Characterizing this subsidy turns out to play a key role in our comparative statics exercises and analysis of policies.

Corollary 1. *Suppose I has previously sold to $N_I \geq 0$ consumers and E has previously sold to $N_E \geq 0$ consumers. In the unique MPE,*

- *if $\Delta(N_I, N_E + 1) \leq s_E - s_I < \Delta(N_I, N_E)$, then I wins and E subsidizes*
- *if $\Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I + 1, N_E)$, then E wins and I subsidizes*
- *otherwise, the losing firm does not subsidize.*

The willingness of the losing firm to offer a subsidy reflects that in the off-equilibrium situation that the losing firm actually wins the current period, it expects to have a positive value (via some future profits). This implies the winning firm's competitive advantage will be limited by the losing firm's willingness to subsidize consumers in such cases. Taking the first case in Corollary 1, it can be divided into two cases. If $\Delta(N_I, N_E + 1) \leq s_E - s_I < \min\left\{\Delta\left(\overline{N}_I, N_E + 1\right), \Delta\left(N_I, N_E\right)\right\}$, then I wins and E subsidizes for a finite number of periods.[9] Meanwhile, if $\Delta(\overline{N}_I, N_E + 1) \leq s_E - s_I < \Delta(N_I, N_E)$, then I wins and E subsidizes

[9] Specifically, E subsidizes for the first k periods, where $k \leq \overline{N}_I - N_I$ is uniquely defined by the inequality $\Delta(N_I + k, N_E + 1) < s_E - s_I < \Delta(N_I + k + 1, N_E + 1)$.

forever, i.e. even after I reaches its learning threshold. And symmetrically when E wins and I subsidizes. Moreover, Corollary 1 does not rule out the winning firm also subsidizing consumers. Specifically, if $f_I(N_I) - f_E(N_E) < s_E - s_I < \Delta(N_I, N_E)$, then I wins and subsidizes in the current period (and symmetrically for E).

In our baseline model we ignored data depreciation to keep things tractable. To incorporate data depreciation, in Online Appendix B, we assume instead that both firms lose a fixed amount of data through depreciation each period, subject to the constraint that the stock of data of each firm cannot fall below zero.[10] For instance, if firm I wins period 1, then the stocks of data at the beginning of period 2 are $\max\{N_E - \rho, 0\}$ and $N_I - \rho + 1$, where $0 \leq \rho \leq 1$ is the fixed amount of data depreciation in each period. With this setup, we show that the threshold for E to win (i.e. I's competitive advantage) becomes

$$\Delta^d(N_I, N_E) = (1-\delta)^2 \sum_{j=0}^{\infty} \sum_{k=0}^{j} \delta^j \left(f_I\left(\max\{N_I - j\rho + k, 0\}\right) - f_E\left(\max\{N_E - j\rho + k, 0\}\right)\right).$$

It is easily verified that $\Delta^d(N_I, N_E)$ converges to $\Delta(N_I, N_E)$ defined in (2) when $\rho \to 0$. When $\rho > 0$, the analysis is more complex due to the double summation, but the model dynamics and results remain similar.

3.1 Determinants of competitive advantage

We are interested in the effects on a firm's competitive advantage of changes in technology that increase the amount the firms can learn from data (e.g. improvements in algorithms) and of shocks that provide firms with more data (e.g. getting access to a new data source that moves them along their learning curves), while taking into account that the incumbent has more data than the entrant.

To explore these types of questions, in this section we put some more structure on the learning functions. Formally, we assume when firm i has sold to N_i past consumers, the value it offers from learning is $f_i(N_i) = \theta_i \max\left\{\min\left\{N_i, \overline{N}_i\right\} - \underline{N}_i, 0\right\}$, where $\underline{N}_i$ and $\overline{N}_i$ are the minimum and maximum thresholds for firm i and $0 \leq \underline{N}_i \leq \overline{N}_i$. This formulation means that the learning curves are S-shaped (see figure 1): the value is zero until some minimum amount of data is collected, after which it increases at a constant rate until learning has been exhausted. It includes as special cases linear learning functions up to the threshold ($\underline{N}_i = 0$), and step learning functions ($\overline{N}_i - \underline{N}_i = 1$ and $\underline{N}_i > 0$).

[10] The difficulty with depreciation proportional to the stock of data in our setting is that the stocks of data after any given number of periods would not just depend on how many periods each firm has won, but also on the sequence of wins/losses for each firm.

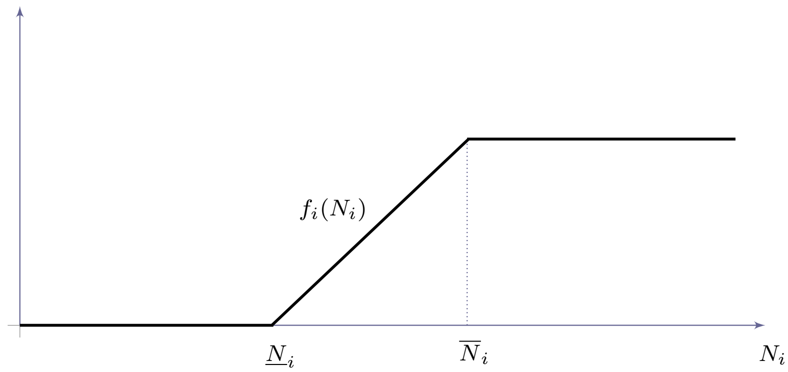

Figure 1

Plugging these functions into (2) leads to the following expression of the cutoff for E to win:

$$\begin{aligned} \Delta(N_I, N_E) \quad &= \quad \theta_I \left(\max\left\{N_I - \underline{N}_I, 0\right\} + \frac{\delta^{\max\{\underline{N}_I - N_I, 0\}+1} - \delta^{\overline{N}_I - N_I + 1}}{1-\delta} \right) \\ & \quad\quad -\theta_E \left(\max\left\{N_E - \underline{N}_E, 0\right\} + \frac{\delta^{\max\{\underline{N}_E - N_E, 0\}+1} - \delta^{\overline{N}_E - N_E + 1}}{1-\delta} \right), \end{aligned} \tag{3}$$

for all $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. Using this cutoff we are able to derive the following comparative statics.

Proposition 2. *Suppose both firms have the same S-shaped learning function with rate of learning θ and thresholds $\underline{N}$ and $\overline{N}$, and assume $N_E < N_I \leq \overline{N}$. Then (i) increasing the value of new data by a multiplicative factor greater than one increases I's competitive advantage when $N_E < N_I \leq \underline{N}$ and decreases I's competitive advantage when $\underline{N} < N_E < N_I \leq \overline{N}$; (ii) increasing both N_I and N_E by $k \geq 1$ increases I's competitive advantage when $N_I + k \leq \underline{N}$ and decreases I's competitive advantage when $N_E > \underline{N}$ and $N_I + k \leq \overline{N}$.*

For (i), increasing the value of new data (relative to the starting point (N_I, N_E)) by a multiplicative factor $\alpha > 1$ means the learning function for firm $i \in \{I, E\}$ changes from $f_i(N_i') = \theta_i \max\left\{\min\left\{N_i', \overline{N}\right\} - \underline{N}, 0\right\}$ to $\widetilde{f}_i(N_i') = f_i(N_i) + \alpha\left(f_i(N_i') - f_i(N_i)\right)$ for all $N_i' \geq N_i$. When the firms have yet to move onto the increasing part of the learning curve, increasing the value obtained from new data in a multiplicative way means it only helps the firms after they start obtaining positive value. As such, it is equivalent to the general multiplicative improvement in learning technology that we discussed earlier. The improvement in technology always benefits the winning firm, which is the incumbent here. Things are different once both firms are already on the increasing part of the learning curve.

Then making new data more valuable means old data is relatively less valuable, which reduces I's competitive advantage, because the latter is based on the value of its old data ($N_I > N_E$). Put differently, E, which has more scope for new learning, benefits more from the improvement in learning with respect to new data.

The first part of (ii) can be interpreted as considering what happens when both firms discover some new data, so that both firms find themselves closer to where they can start to offer positive value from learning. This discovery increases I's competitive advantage. To understand this result note that if firms are yet to move onto the increasing part of the learning curve, requiring fewer periods of customer data to reach the increasing part brings forward (i.e. discounts less) the advantage that I obtains from being closer to the threshold, thereby increasing its competitive advantage. The effect reverses when both firms are already on the increasing part of the learning curve (second part of (ii)). Here, a reduction in the number of learning periods both firms need to reach the maximum threshold decreases the number of periods over which I can enjoy its learning advantage, so helps E.

3.2 Welfare analysis

A common counterfactual used to assess the efficiency of the entrant making sales is to think about who would win if the entrant had access to the same data as the incumbent. Obviously, in a dynamic setting, it also matters how much value consumers get along the path to such an outcome, which is why the correct approach involves looking at the PDV of gross surplus along respective winning paths starting from the current data that each firm has. Specifically, if I wins in every period, the value created is $\frac{s_I-c}{1-\delta}+\sum_{j=0}^{\infty}\delta^j f_I(N_I+j)$, whereas if E wins in every period, the value created is $\frac{s_E-c}{1-\delta}+\sum_{j=0}^{\infty}\delta^j f_E(N_E+j)$. The socially optimal cutoff is then the level of s_I-s_E for which these two expressions are equal, which is easily seen to exactly correspond to the equilibrium cutoff $\Delta(N_I,N_E)$ from (2). We have thus proven the following result.

Proposition 3. *The unique MPE outcome characterized in Proposition 1 is socially optimal.*

The logic behind this efficiency result is actually quite subtle. To understand this, consider first a two-period model, in which the firms start with (N_I,N_E) respectively and there is only one opportunity to learn, arising from the opportunity to win consumers in the first period. To determine which firm wins in the first period, note that the maximum surpus that each firm can afford to offer consumers in the first period is its period 1 surplus plus the discounted value of the additional surplus that it offers in period 2 relative to the other firm. Thus, E wins in period 1 iff

$$\begin{aligned} & s_E + f_E(N_E) + \delta\left(s_E + f_E(N_E+1) - s_I - f_I(N_I)\right) \\ \geq \quad & s_I + f_I(N_I) + \delta\left(s_I + f_I(N_I+1) - s_E - f_E(N_E)\right). \end{aligned}$$

By contrast, the efficient condition for E to win is simply

$$\begin{aligned} & s_E + f_E(N_E) + \delta\left(s_E + f_E(N_E+1)\right) \\ \geq \quad & s_I + f_I(N_I) + \delta\left(s_I + f_I(N_I+1)\right). \end{aligned}$$

Comparing the two inequalities above, the competitive outcome is inefficient due to the double-counting of the respective first-period surpluses, $s_E + f_E(N_E)$ and $s_I + f_I(N_I)$. This means that despite the simple Bertrand nature of competition, it is not necessarily the firm that offers the largest PDV of surplus that wins in the first period (which is what simple intuition would suggest). In other words, the condition for E to win places too much weight on first-period surpluses relative to second period surpluses, because the first-period surplus offered by the losing firm also reduces the profit extracted by the winning firm in period 2.

This inefficiency will be present in the setting with any finite number of periods $T \geq 2$. Indeed, in Online Appendix C we provide the condition for E to win for all $T \geq 2$ and show that this condition is generally inefficient, with too much weight placed on earlier periods and too little weight placed on later periods. Finally, we also show that the condition for E to win converges to the efficient condition given in Proposition 1 when T tends to infinity.

The reason that the inefficiency fades away as T goes to infinity is that the same type of double-counting spreads out across all periods, in the limit becoming proportional to the socially efficient way of accounting for the surplus offered by each firm in each period along its winning path.

The PDV of consumer surplus is determined by the surplus offered by the losing firm in each period along the equilibrium path (given the same firm wins in all periods). As a result, learning by the winning firm as it sells to additional consumers each period makes consumers weakly worse off. To see this note first that if the losing firm has already stopped offering a subsidy, then learning by the winner does not change the PDV of consumer surplus offered by the loser, which in case firm j is the losing firm is fixed at

$$CS(N_I, N_E) = \frac{s_j - c + f_j(N_j)}{1-\delta}. \tag{4}$$

In case the losing firm currently offers a subsidy, the PDV of consumer surplus can be determined by the PDV of total surplus less the PDV of the winning firm's profit stream, i.e.

$$CS\left(N_I, N_E\right) = \frac{s_i - c}{1-\delta} + \sum_{k=0}^{\infty} \delta^k f_i(N_i + k) - \left(\frac{s_i - s_j + \Delta(N_I, N_E)}{(1-\delta)^2}\right) \tag{5}$$

in case firm i is the winning firm and firm j is the losing firm. It is straightforward to check that in this case $CS\left(N_I, N_E\right)$ is decreasing in N_i, so consumer surplus is strictly decreasing as the winning firm learns more along the equilibrium path. This reflects that the losing firm's subsidy gets smaller over time as the winning firm's learning advantage continues to increase along the equilibrium path.

An inspection of the PDV of payoffs for firms from the proof of Proposition 1, and of consumers in (4) and (5), reveals that the consumers' and winning firm's payoffs move in opposite directions with respect to the key drivers of competitive advantage. More formally, we have the following proposition.

Proposition 4. *When firm i wins and firm j loses, consumer surplus is increasing in s_j and $f_j\left(N_j\right)$, and weakly decreasing in s_i and $f_i\left(N_i + k\right)$ for all $k \geq 0$. Meanwhile, the winning firm i's profits are decreasing in s_j and $f_j\left(N_j\right)$, and increasing in s_i and $f_i\left(N_i + k\right)$ for all $k \geq 0$.*

In a more general model of competition with horizontal differentiation between firms or in a setting with elastic aggregate demand, we may expect some of the winning firm's learning benefit to be shared with consumers. However, the result that consumers can be made worse off by the winning firm's learning because the losing firm is no longer willing to subsidize as much in an attempt to compete is likely to be much more general. It implies consumers can be better off if they could somehow deviate by coordinating on joining the losing firm instead of the winning firm for a number of periods. The problem is that, given each individual consumer is atomless and can freely switch in each period, she has no reason to internalize this effect as her own decision about which firm to join will not affect the consumer surplus she can obtain next period (in the case she is long lived). This suggests a possible role for a data sharing policy, which we consider in the next section.

3.3 Data policy

In this subsection we use our framework to examine the effect on competition and consumer welfare of three possible types of data policies: (i) requiring data sharing, (ii) enforcing stricter consumer data privacy rules, and (iii) preventing an incumbent firm from acquiring a non-competing firm that contains valuable data.

3.3.1 Data sharing

Requiring the incumbent to share some of its data with the entrant would be one way to help the entrant catch up to the learning of the incumbent, and based on our earlier welfare analysis, this would seem to be good for consumers. Indeed, this would trivially be the case for a policy in which the losing firm got to share any data of the winning firm as a one-off intervention. However, if the incumbent's data is always shared with the entrant, then it may dampen both firms' incentives to invest in building up their data in the first place. In our framework, such investments are endogenously determined by the extent of subsidies offered to consumers. Specifically, once the firm that is behind is able to share in the other firm's data through a data sharing policy, it will compete less aggressively (a form of free-riding), which can result in higher prices and therefore lower consumer surplus.

To show this tradeoff formally, we consider the introduction of a policy which requires any *new* data collected by firms be shared with each other. In our model, in every period subsequent to the implementation of data sharing, both firms advance their learning by one period (if they are not already at the threshold) regardless of who wins. This means that from the period the policy is implemented onwards, which we treat as being in the first period, the lowest price each firm is willing to charge is c. In every period $j \geq 0$, firm I wins if $s_I + f_I(N_I + j) > s_E + f_E(N_E + j)$, and firm E wins otherwise. Consequently, the PDV of consumer surplus after the implementation of the data sharing policy is

$$CS(N_I, N_E)' = \sum_{j=0}^{\infty} \delta^j \min\{s_I + f_I(N_I + j), s_E + f_E(N_E + j)\} - \frac{c}{1-\delta}. \tag{6}$$

Meanwhile, in the absence of the data sharing policy, when $s_E - s_I < \Delta(N_I, N_E + 1)$ or $s_E - s_I \geq \Delta(N_I + 1, N_E)$, the losing firm does not subsidize so the PDV of consumer surplus is just the PDV of the surplus offered by the losing firm, as in (4). When $\Delta(N_I, N_E + 1) \leq s_E - s_I < \Delta(N_I + 1, N_E)$, the losing firm subsidizes, and the PDV of consumer surplus absent data sharing is given in (5).

Comparing the expressions for consumer surplus, we obtain the following proposition (the proof is in the appendix).

Proposition 5. *Suppose data sharing requires whichever firm wins a given period to share the data obtained with its rival. Suppose one firm is closer to its learning threshold than the other, i.e.* $\overline{N}_I - N_I < \overline{N}_E - N_E$ *or vice versa. Then there exist cutoff levels* $\Theta_l(N_I, N_E) < \Delta(N_I, N_E)$ *and* $\Theta_h(N_I, N_E) > \Delta(N_I, N_E)$ *such that:*

- *Data sharing increases the PDV of consumer surplus when* $s_E - s_I \leq \Theta_l(N_I, N_E)$ *or* $s_E - s_I \geq \Theta_h(N_I, N_E)$

- *Data sharing decreases the PDV of consumer surplus when* $\Theta_l(N_I, N_E) < s_E - s_I < \Theta_h(N_I, N_E)$.

Thus, data sharing increases consumer surplus when the losing firm is at a sufficiently large disadvantage and decreases consumer surplus when the two firms are sufficiently evenly matched. This is intuitive. When the losing firm is at a large disadvantage, it does not subsidize, so there is no downside from data sharing, and therefore the positive effect of keeping the firms evenly matched in all future periods dominates. However, when the two firms are more evenly matched, the losing firm would normally subsidize heavily and data sharing removes this incentive to subsidize, so this is where the downside of data sharing is highest and overwhelms the positive effect.

Although consumers may be better or worse off under data sharing, it is easily seen that the PDV of total welfare is higher with this type of data sharing because in every period $j \geq 0$, the total surplus created is $\max\{s_I + f_I(N_I + j), s_E + f_E(N_E + j)\} - c$. In other words, both firms are learning in every period so society gets the best of both in every period.

3.3.2 Privacy of consumer data

A stricter privacy policy makes it harder for firms to gather customer data either by restricting the type of data they can make use of or by making it easier for consumers to opt out of sharing their data. We model this as both firms only being able to use a fraction of any *new* customer data collected once the privacy policy has been implemented, assuming the learning curves are S-shaped as in (3). In other words, the linear portion of each firm i's learning curve pivots to the right from its current position, which represents a slowing down of the learning rate from customer data. However, firm i still eventually reaches the same maximum value from learning of $f_i\left(\overline{N}_i\right)$. This is illustrated in figure 2, where the learning curve f_i pivots to f_i' after the privacy policy is implemented.

The following proposition determines the effect of such a privacy policy on I's competitive advantage and on consumer surplus.

Proposition 6. *Suppose both firms have S-shaped learning functions. Suppose I is (weakly) closer to reaching its threshold and E's learning algorithm is at least as good (i.e.* $\overline{N}_I - N_I \leq \overline{N}_E - N_E$ *and* $\theta_E \geq \theta_I$*, with one inequality strict), both firms have already obtained some positive value from learning (i.e.* $N_I \geq \underline{N}_I$ *and* $N_E \geq \underline{N}_E$*), and I would win absent the policy change (i.e.* $s_E - s_I < \Delta(N_I, N_E)$*). Then a privacy policy that slows down the rate*

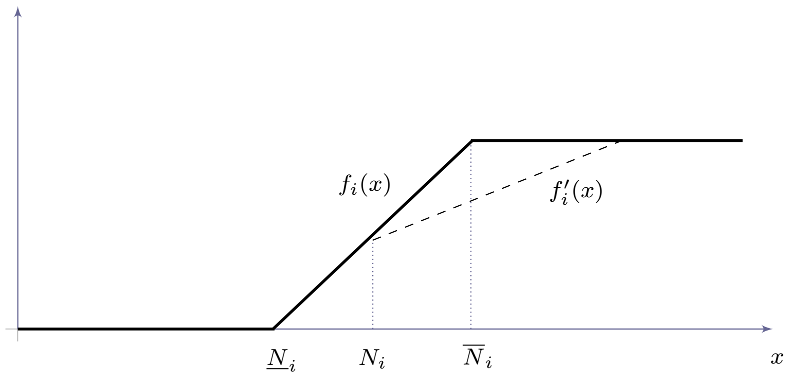

Figure 2

at which both firms learn from their customers' data increases I's competitive advantage and weakly decreases consumer surplus, strictly decreasing it if E subsidizes consumers before the policy change.

The effect on I's competitive advantage means that, when I is initially closer to its threshold and/or E has a steeper learning function, slowing down the rate of learning has a disproportionately negative effect on E. This reflects that E has more to gain from new customer data, and therefore more to lose from reductions in the amount of data that is usable.[11] The logic of this effect is easiest to see in the extreme case when I is at (or almost at) its threshold, and E is far away from reaching its threshold. A privacy policy that slows down the amount of useful data that both firms can extract from their customers will have no (or almost no) effect on I, as it has nothing (or almost nothing) left to learn, but it can have a big effect on E which has a lot left to learn.

In contrast to the general result we discussed after Proposition 1 that the incumbent (or more generally the winning firm) benefits from an improvement in learning technology, here the incumbent benefits from a worsening of the technology. This reflects that here the change in learning technology (i.e. slower learning) only applies to new data obtained after the change, with the same eventual upper threshold for learning, whereas a general improvement in technology applies to learning from existing as well as new data, with the upper threshold for learning also changing. Thus, the effect of slowing down learning (by implementing a privacy policy) is consistent with the effect of worsening the value of learning from new data when both firms are on the increasing part of the S-shaped learning curve, that we determined in part (i) of Proposition 2. The difference is that here the learning

[11]The result no longer holds generally when either E or both I and E are yet to move onto the increasing parts of their learning curves. However, one can show it still holds in that case provided firms do not discount the future too much.

threshold stays unchanged, whereas a proportional worsening of the learning from new data also reduces the learning threshold.

The decrease in consumer surplus in Proposition 6 arises because E lowers the subsidy it offers, reflecting that it has less to gain by winning after the (stricter) privacy policy is introduced. As a result, I (which continues to win given its competitive advantage increases) doesn't have to leave as much surplus with consumers to prevent them from switching to E. Although we haven't modeled any direct consumer benefits due to tighter privacy rules, any such benefits should not affect the result given consumers can internalize them when deciding which firm to join. We are, of course, abstracting from externalities (positive or negative) that the use of a consumer's private data may impose on other consumers or firms. Thus, our result is not meant to speak to the overall effects of a stricter privacy policy, but to point out that there may be an unintended downside of imposing stricter privacy policies on competitive dynamics in general, and entrants with less customer data in particular.

3.3.3 Killer data acquisitions

Our framework can also be used to explore the interesting question of what happens when firms can acquire an outside firm (or its dataset) in order to bolster their data position. Starting from (N_I, N_E), assume there is an opportunity for E and I to acquire a measure $N_A > 0$ of data exclusively. We wish to know which firm will end up acquiring the data, which is equivalent to figuring out which firm is willing to pay more for it (assuming the owner of the data can commit to sell it exclusively), and whether the outcome will be efficient.

For each firm, there are potentially two sets of incentives to acquire data: one to create new rents by using the data to move up the learning curve, and the other to protect rents by denying valuable data to the competitor. For instance, if the incumbent has already reached or is closer to its learning threshold than the entrant, but the entrant has a superior learning technology, then the incumbent's incentive is primarily to protect rents, and the entrant's is to deploy the data via its superior technology. We formalize this idea in what follows.

If I acquires N_A, then the cutoff for E to win increases to $\Delta(N_I + N_A, N_E)$, whereas if E acquires N_A, then the cutoff for E to win decreases to $\Delta(N_I, N_E + N_A)$, with the understanding that $\Delta(N_I + N_A, N_E) = \Delta\left(\overline{N}_I, N_E\right)$ whenever $N_I + N_A \geq \overline{N}_I$ and $\Delta(N_I, N_E + N_A) = \Delta\left(N_I, \overline{N}_E\right)$ whenever $N_E + N_A \geq \overline{N}_E$. We will use this slight abuse of notation to state our result in the following proposition in order to keep the expressions compact.

Proposition 7. *Suppose there is an opportunity for E or I to acquire a dataset of measure $N_A > 0$. Assume at least one of the two firms (I or E) has not reached its maximum learning* *threshold. Then there exists a unique* Δ^* *satisfying*

$$\Delta\left(N_I, N_E + N_A\right) < \Delta^* < \Delta\left(N_I + N_A, N_E\right) \tag{7}$$

such that I wins the data if and only if $s_E - s_I < \Delta^*$*, and E wins the data otherwise. If* $\Delta^* < \Delta\left(N_I + N_A, N_E + N_A\right)$*, then the cutoff in* $s_E - s_I$ *above which E acquires the data is too low from a social efficiency perspective (i.e. E acquires the data too often), whereas if* $\Delta^* > \Delta\left(N_I + N_A, N_E + N_A\right)$*, then in equilibrium I acquires the data too often.*

As shown in the proof in the Appendix, both scenarios in Proposition 7 are possible. Thus, the equilibrium outcome of the data acquisition game can be socially inefficient in either direction. Contrast this with our general efficiency result for the baseline model from Proposition 3. The reason for the difference is that here the competition is for buying data from a third party, who only cares about price and not about the respective utilities currently offered by the two firms. By contrast, when the two firms compete for consumers, these consumers care about the current utility and the price they are being charged.

To illustrate this result, we can look at a special case. Suppose one of the firms—I without loss of generality—is already at the threshold, i.e. $N_I = \overline{N}_I$. Then we have

$$\Delta\left(N_I + N_A, N_E + N_A\right) = \Delta\left(\overline{N}_I, N_E + N_A\right) = \Delta\left(N_I, N_E + N_A\right) < \Delta^*,$$

where the last inequality follows directly from (7). In this case I wins the data too often. And conversely when E is already at the threshold. In other words, in the equilibrium of the acquisition game when one firm has already reached its threshold of learning, that firm wins too often relative to what is socially efficient. The idea is that the amount the firm that is at the threshold gives up by not winning (namely its profit when competing with a rival that is below its learning threshold) is greater than the amount that the rival can gain by acquiring the data and winning, as in this case it competes with a rival that is already at its threshold. In terms of the two firms' incentives, I's rent protection motive dominates. Thus, we provide a theory of killer data acquisitions, in which the firm acquiring the data has no use for it, and only acquires it to deny the rival access to it.

A final property of interest is that even though I may acquire the new data in equilibrium, it doesn't necessarily benefit from it. The existence of the new data N_A makes it easier for E to catch up, and I needs to buy the data to prevent E catching up, so I can end up worse off as a result of the existence of the new data.[12]

[12] We show this result formally in Online Appendix D.

3.4 Within-period learning

So far consumers have not had to form beliefs about which firm other consumers will choose in any given period. Although there is a self-reinforcing dynamic across periods in which a firm that attracts consumers this period improves its product and finds it easier to attract consumers in subsequent periods, current consumers still did not face the usual coordination problem that arises under classic network effects. Indeed, as noted earlier, even with long-lived consumers, because each consumer was atomistic and could costlessly switch each period, a consumer's decision in one period had no effect on the opportunities they could enjoy in subsequent periods, so there was no reason for any consumer to take into account the decisions of other contemporary consumers.

In this section we show that a consumer coordination problem arises when across-user learning is combined with the assumption that the product continues to improve during the period(s) over which consumers derive utility from the product after paying for it. To do so, we modify the baseline model by allowing firms to learn immediately from consumers purchasing in the current period, rather than learning next period only, as we had assumed previously. As argued in the introduction, within-period learning is a key distinguishing feature of data-enabled learning in the context of cloud-based products, and contrasts with learning in the context of traditional products, where the new and improved features were only embedded in the next releases of the product, which would be sold to future consumers. Because of this feature, other things equal, consumers prefer to buy from the firm they expect others to also buy from, given they benefit from the resulting improvement in the firm's product during the consumption period.[13]

We consider consumers resolving the resulting coordination problem via two different types of beliefs. First, we assume that in every period, consumers believe all other consumers will coordinate on the equilibrium outcome that is best for them in that period, taking into account the prices they face. This selects the equilibrium which maximizes the consumers' surplus (i.e. net utility) within every period. When consumers live for one period only, which recall is one interpretation of our across-user learning setup, this corresponds to the Pareto optimal equilibrium for consumers for given prices, and with this in mind, we refer to these beliefs as "Pareto beliefs". We then contrast Pareto beliefs with beliefs that always favor the incumbent. The latter means that in each period, consumers believe all other consumers will buy from I whenever this is an equilibrium outcome given the prices charged by the two firms.

Relative to Proposition 1, the case of within-period learning and Pareto beliefs is as if

[13] As is standard in models with network effects, this assumes there is some friction so that consumers all choose simultaneously within a given period.

both firms were one period closer to their respective maximum learning thresholds, whereas the case of within-period learning and beliefs favoring I is as if only I were one period closer to its maximum learning threshold. Then following an almost identical proof to that of Proposition 1 we obtain the following result.

Proposition 8. *Suppose I has previously sold to $N_I \geq 0$ consumers and E has previously sold to $N_E \geq 0$ consumers. With Pareto beliefs and within-period learning, E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I+1, N_E+1)$, and I wins in all periods otherwise, where $\Delta(N_I, N_E)$ is defined by (2). With favorable beliefs for I and within-period learning, E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I+1, N_E)$, and I wins in all periods otherwise. The equilibrium outcome is socially efficient with Pareto beliefs and involves I winning for a larger range of $s_E - s_I$ than is socially efficient under favorable beliefs for I.*

The intuition for these results is the same as before, adjusted for the fact that the winning firm now learns from its current period's consumers. Note the socially optimal outcome takes into account within-period learning, just as the equilibrium conditions do. Moreover, Corollary 1 continues to apply, given that the cutoffs that apply here are defined in the same way as (2), but with the terms in N_I and N_E increased by zero or one depending on the type of beliefs. By the same logic, all other results similarly carry over.

Taking the difference in cutoffs when beliefs favor I vs. when beliefs are Pareto, we obtain

$$\Delta(N_I+1, N_E) - \Delta(N_I+1, N_E+1) = (1-\delta)\left(\sum_{j=0}^{\overline{N}_E - N_E - 1} \delta^j \left(f_E(N_E+j+1) - f_E(N_E+j)\right)\right), \tag{8}$$

which is strictly positive whenever $N_E < \overline{N}_E$.[14] The expression in (8) is a measure of the distortion that arises in the cutoff for E to win due to beliefs favoring I, relative to the socially optimal outcome. It shows I's position relative to its learning threshold is irrelevant to the distortion implied by I enjoying favorable beliefs. This is because under both types of beliefs, consumers evaluate the PDV of future product improvements that can be offered by I assuming I will win in every period. Meanwhile, E is effectively viewed by consumers as being one period of learning behind in every period under beliefs that favor I compared to under Pareto beliefs.

Taking the difference between (8) evaluated at $N_E - k$ and (8) evaluated at N_E, the following result follows almost immediately.

[14]When $N_E = \overline{N}_E$, beliefs are irrelevant, and (8) is equal to zero.

Proposition 9. *Suppose there is across-user learning and within-period learning. If E's learning function f_E is weakly concave, the distortion in the cutoff resulting from beliefs favoring I is higher the further away E is from its maximum learning threshold (i.e. the lower N_E is).*

This result reflects that shifting E back by one period of learning has a bigger effect when E is on the steeper (i.e. earlier) part of its learning function, and that the corresponding disadvantage for E arises over additional periods when E is further away from its maximum learning threshold. Put more directly, under the conditions of Proposition 9, the concern that beliefs favoring I give I an inefficient advantage is greater the further away E is from its maximum learning threshold.

4 Within-user learning

In this section we focus on within-user learning. We assume the same measure one of forward-looking consumers that are present in every period. They are infinitely lived and can switch firms in any period. The value to a given consumer of firm i's product in the current period is $s_i + f_i(N_i)$ for $i \in \{I, E\}$, where $f_i(N_i) = f_i\left(\overline{N}_i\right)$ for all $N_i \geq \overline{N}_i$. The main difference from the case with across-user learning is that here N_i refers to the number of times the consumer has purchased from the same firm before, rather than the total number of consumers that the firm has sold to. Another key difference is that in the presence of within-user learning, it is natural to allow firms to price discriminate across consumers that have different histories. Unlike the case with across-user learning, here the individual consumer's history affects the value offered by each firm's product. Consistent with our perfect information setup, we assume that both firms can observe how many periods a customer has consumed from each firm previously, and can price discriminate accordingly.[15] As there is no across-user learning and firms can price discriminate with respect to individual consumers, the analysis can be done independently for each individual consumer. Viewed in this light, the model is consistent with I and E being in different competitive positions with respect to different individual consumers (or consumer segments), and our results pertain to any such consumers (or consumer segments).

[15] Obviously, each firm will know how many times a consumer has purchased from it before, so can condition its prices on this information. Given the model setup in which the same consumers are present in every period, each firm would therefore also be able to work out how many times a consumer has purchased from the rival firm in the past. In practice, firms may elicit this information from consumers by requiring a consumer supply their past purchase records so as to provide them with a better deal if they switch.

The following proposition characterizes when E wins, when I wins, the social efficiency of the outcome, and the value obtained by firms.

Proposition 10. *Suppose I has sold $N_I \geq 0$ times in the past to a given consumer and E has sold $N_E \geq 0$ times to that consumer. Then a unique MPE exists in which E wins in all periods if and only if $s_E - s_I \geq \Delta(N_I, N_E)$, and I wins in all periods otherwise, where $\Delta(N_I, N_E)$ is defined by (2). The outcome is socially optimal. Moreover, the conditions for the losing firm to subsidize are identical to those in Corollary 1.*

Proposition 10 shows that the cutoff in $s_E - s_I$ that determines which firm wins in each period coincides with that obtained in the across-user learning case, and remains socially optimal.[16] This means our comparative static results carry over to the case of within-user learning. However, despite this equivalence, the logic behind the result and its proof are quite different. With across-user learning, we never had to solve the consumers' forward-looking problem given their choice in any given period didn't affect their options in subsequent periods. In contrast, here consumers have to take into account the effect of their current choice of firm on their future options (within-user learning creates a switching cost), and so we need to determine the value functions for consumers as well as for firms. In the proof of Proposition 10, we construct the consumers' value functions, and show that consumers never expect to switch firms in equilibrium.

It is worth pointing out that Proposition 10 and its implications for comparative statics can be applied at the level of an individual consumer (or consumer segment). Doing so for a continuum of different consumers implies that I would sell in each period to consumers with high values of s_I and E would sell in each period to consumers with high values of s_E. On the surface the market might not look monopolized at all, but eventually each firm might face little or no competitive pricing pressure from its rival as its data advantage with respect to its own customers becomes sufficiently large. This suggests competition authorities should pay more attention to the length of customer retention rather than asymmetric market shares in such settings.

In contrast to the case of across-user learning, the logic for why the equilibrium outcome under within-user learning coincides with the socially optimal outcome is more straightforward. Here consumers are forward-looking, so they take into account all future subsidies and utility they will receive from their chosen firm. Starting from any point, whichever firm creates greater PDV of gross surplus from learning, assuming it wins every period, will be able to offer a higher level of utility plus subsidy to consumers, and so will win in the current

[16] It is easily confirmed, the results in Proposition 10 remain valid when $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$, provided learning functions are bounded by some power function.

period given the Bertrand setting. And if a particular firm wins in the current period, this just further increases the value it can offer in future periods, thereby ensuring the same firm wins in all future periods. Consistent with this, in Online Appendix E we confirm that the outcome under within-user learning and a finite time horizon is also always socially optimal, in contrast to what we found with across-user learning.

In the case of within-user learning, outcomes are more competitive (e.g. the winning firm's profit is strictly lower) than with across-user learning for the same initial data and learning functions.[17] The logic is that within-user learning creates an increasing switching cost: consumers take into account that the surplus they will be left with in any period is the surplus offered by the losing firm (due to Bertrand competition), and that this surplus will be lower if they buy from the winning firm this period versus if they buy from the losing firm this period. Thus, the winning firm must compensate consumers by pricing lower in the current period compared to the equivalent case with across-user learning.[18] This result would no longer hold if consumers were myopic. Indeed, the setting with within-user learning but myopic consumers would result in exactly the same outcome as the case with across-user learning, where consumers had no reason to consider more than their current surplus when deciding which firm to buy from.

We now turn to the data policy issues analyzed earlier with across-user learning. First, the effect of unanticipated data sharing on consumer surplus works in a very similar way to the case with across-user learning. The PDV of consumer surplus after the implementation of the same type of policy we considered earlier is the same here, equal to the expression in (6):

$$CS\left(N_I, N_E\right)' = \sum_{j=0}^{\infty} \delta^j \min\left\{s_I + f_I(N_I + j), s_E + f_E\left(N_E + j\right)\right\} - \frac{c}{1-\delta}.$$

Meanwhile, the PDV of consumer surplus without the data sharing policy is

$$CS\left(N_I, N_E\right) = \begin{cases} \frac{s_E - c}{1-\delta} + \sum_{j=0}^{\infty} \delta^j f_E(N_E + j) & \text{if} \quad s_E - s_I < \Delta(N_I, N_E) \\ \frac{s_I - c}{1-\delta} + \sum_{j=0}^{\infty} \delta^j f_I(N_I + j) & \text{if} \quad s_E - s_I \geq \Delta(N_I, N_E) \end{cases}.$$

And it is easily seen that $CS\left(N_I, N_E\right)' \leq CS\left(N_I, N_E\right)$ for all (N_I, N_E), which means that data sharing always reduces consumer surplus here, in contrast to the case with across-user learning, where data sharing only reduced consumer surplus when $s_E - s_I$ was in a certain interval. The fact that data sharing is more likely to reduce consumer surplus under within-

[17]This result follows from a straightforward comparison of the value functions from the proof of Proposition 1 with the value functions from the proof of Proposition 10. This is spelled out formally in Online Appendix F.

[18]This result bears some resemblance to Section 5 in Taylor (2003), where consumers draw different switching costs (high or low) that are unknown to firms. In his model, consumers anticipate being held up if they are revealed to have high switching costs, so must be compensated for purchasing from the same firm.

user learning is consistent with the result above that within-user learning makes the two firms compete more aggressively in the absence of data sharing, which diminishes the potential benefits of data sharing in the first place.

Second, the effect of a stricter privacy policy on I's competitive advantage is the same as with across-user learning because the cutoff $\Delta(N_I, N_E)$ is the same here. The expression of the change in consumer surplus due to the implementation of the stricter privacy policy is different, but it turns out by following a very similar proof, to also be negative. Thus, under the same conditions as Proposition 6 but applied to within-user learning, a stricter privacy policy again lowers consumer surplus.

Third, and finally, the reasoning for and effects of data acquisition are the same here relative to the case with across-user learning. In particular, Proposition 7 remains valid, even though the exact expression for the equilibrium cutoff Δ^* that determines which firm wins the acquisition game is different.

5 Across-user and within-user learning

The purpose of this section is to briefly explore the implications of combining across-user and within-user learning; in particular how doing so can sometimes lead to an endogenous network effect in which consumer beliefs matter.

To show this, we combine the models of Sections 3 and 4 in the following way. First, we assume there is just one fixed set of consumers of measure one who are present in all periods, consistent with the model of within-user learning used in Section 4. And second, we assume that the value a given consumer obtains from firm i's product in the current period depends on the two types of learning—the number of times that this measure one of consumers has previously purchased from the same firm (across-user learning), and the number of times the same consumer has previously purchased from the same firm (within-user learning).

A consumer who has previously consumed n_i times from firm $i \in \{I, E\}$ when firm i has previously served the measure one of consumers N_i times in the past derives utility $s_i + f_i(N_i, n_i)$ in the current period if she chooses firm i's product. We assume $f_i(N_i, n_i) = f_i(\overline{N}_i, n_i)$ for all $N_i \geq \overline{N}_i$ and $f_i(N_i, n_i) = f_i(N_i, \overline{n}_i)$ for all $n_i \geq \overline{n}_i$, where $\overline{N}_i \geq 1$ is the threshold level of across-user learning and $\overline{n}_i \geq 1$ is the threshold level of within-user learning for firm i. We also assume the learning function $f_i(.,.)$ is everywhere weakly increasing in both arguments for $i \in \{I, E\}$. Without loss of generality, we assume that $f_i(0, 0) = 0$.

First, it is straightforward to show that if consumers are myopic, so they only consider their current period payoffs in making their decisions, we can apply all the analysis of the pure across-user learning model from Section 3. To do so, we just need to assume that the initial state is such that the number of times each consumer has chosen firm i in the past corresponds to the number of times the measure one of all consumers have chosen firm i in the past, so we start with $n_i = N_i$. We can then apply our baseline analysis with across-user learning by defining the learning function for firm i as

$$\widetilde{f}_i(N_i) = f_i(N_i, n_i)$$

so it combines both across-user learning and within-user learning, and the maximum learning threshold for firm i as $\max\left\{\overline{N}_i, \overline{n}_i\right\}$. For instance, with these assumptions, Proposition 1 continues to hold[19] provided we replace f_i in the proposition with $\widetilde{f}_i$. Given all the results of Section 3 continue to hold, a researcher that wants to study a setting with across-user and within-user learning can do so using our analysis in Section 3 provided they use the learning function and maximum threshold defined above, and are willing to assume that consumers are myopic.

If on the other hand consumers are forward-looking, things quickly become much more complicated. As such, we focus on the simplest possible case for which consumers can face a coordination problem in which their beliefs matter. Specifically, we assume $\overline{n}_I = \overline{n}_E = \overline{N}_I = \overline{N}_E = 1$ (both firms only need to attract measure one of consumers for one period to achieve maximal learning), $n_I = N_I = 1$ (I has already achieved its maximum learning), and $n_E = N_E = 0$ (E starts with no learning).

To analyze this case, suppose first that consumers hold Pareto beliefs. This means that when consumers are in state $(N_I, n_I, N_E, n_E) = (1, 1, 0, 0)$, i.e. I has benefitted from one period of both types learning whereas E has not had any learning whatsoever, and the prices charged by the two firms in the current period are p_I and p_E respectively, consumers choose I in the current period if and only if

$$s_I + f_I(1,1) - p_I + \delta u(1,1,0,0) > s_E - p_E + \delta u(1,1,1,1),$$

and they choose E otherwise. In other words, consumers coordinate on choosing the firm that offers them the highest PDV of utility. It is then straightforward to show (the proof is in Online Appendix H) that with Pareto beliefs, E wins all periods starting from state $(1, 1, 0, 0)$ iff

$$s_E - s_I \geq f_I(1,1) - \delta f_E(1,1),$$

which is the socially efficient cutoff for E to win.

Suppose instead consumers hold beliefs favorable to I. This means that when consumers

[19] We formally prove this in Online Appendix G.

are in state $(1,1,0,0)$, consumers choose I in the current period if and only if

$$s_I + f_I(1,1) - p_I + \delta u(1,1,0,0) \geq s_E - p_E + \delta u(1,1,0,1),$$

and they choose E otherwise. Note the difference with the case of Pareto beliefs. Here, consumers choose E if and only if it is individually rational for a consumer to choose E even when she expects all other consumers to choose I. This means that to determine the equilibrium we need to characterize what happens in the state $(1,1,0,1)$ of a single consumer that has chosen E for one period whereas everyone else has always chosen I.[20]

By doing so, in Online Appendix H, we are able to prove the following result:

- If $f_E(0,1) \geq \delta f_E(1,1)$, then the cutoff for E to win under beliefs favorable to I is the same as the cutoff under Pareto beliefs, which is equal to the efficient cutoff $f_I(1,1) - \delta f_E(1,1)$.
- If $f_E(0,1) < \delta f_E(1,1)$, then the cutoff for E to win under beliefs favorable to I is higher than the cutoff under Pareto beliefs.

The fact that Pareto beliefs lead to the efficient cutoff is consistent with earlier results, showing that efficiency arises when firms compete in prices. It is also not surprising that E has a harder time winning when beliefs favor I than under Pareto beliefs. The interesting part of the result is that for the outcomes under the two types of beliefs to be different, so that a coordination problem arises, across-user learning must be sufficiently important relative to within-user learning (the condition $f_E(0,1) < \delta f_E(1,1)$).

6 Conclusion

Data-enabled learning is becoming an essential ingredient to understanding competition between providers of an increasing range of products and services. We offer a tractable framework in which to study the effect of various factors on a firm's competitive advantage in the presence of data-enabled learning. Our analysis shows that even when an incumbent starts with a data advantage, if the entrant's learning curve is steeper and can ultimately reach a higher threshold of learning value (possibly due to better technology), through dynamic pricing the entrant may be able to seize the market. Although as expected, an improvement in one firm's learning technology or data always helps that firm, the effect of a common shock to technology or data is less obvious. We find the firm which starts off with more data

[20]The state $(1,1,0,1)$ means I has benefited from both types of learning, whereas E has only benefited from within-user learning for the focal consumer.

(the incumbent) sees its competitive advantage increase when both firms gain access to a better technology (algorithm). Meanwhile, if both firms gain access to the same amount of additional data, this favors the entrant when the firms have already started learning, as a reduction in the number of periods both firms need to reach the maximum threshold level of learning reduces the number of periods over which the incumbent can enjoy its learning advantage. This result reverses when the firms have yet to start learning. Moreover, having access to a concave learning technology is better than a convex learning technology, other things equal. User beliefs favoring the incumbent can be an additional source of competitive advantage for the incumbent, but only if across-user learning is combined with either (i) continued product improvement during the period(s) over which consumers derive utility from the product, or (ii) within-user learning.

We have also derived implications for public policy. A data-sharing policy which forces the winning firm to share the data it obtains with the losing firm in all subsequent periods would seem desirable because it results in increased competitive pressure on the winning firm. However, the anticipation of data-sharing also disincentivizes the losing firm from subsidizing (a form of free-riding). Thus, on balance, data-sharing is good for consumers only when the losing firm is sufficiently far behind the winning firm (which is when it does not have an incentive to subsidize even absent data-sharing). By slowing down the rate at which firms learn, an unintended consequence of stricter privacy rules is that they enhance an incumbent's competitive advantage (because the incumbent is closer to its learning threshold) which also reduces consumer surplus. The framework also has implications for merger policy: it can explain why an incumbent which has already accumulated a lot of data may have an excessive incentive to acquire a firm that contains a lot of data even if it doesn't compete with it or need the data to learn itself. This is driven by the incumbent's interest in stopping rivals from acquiring the firm and learning from its data, which would allow them to become closer competitors, thus providing a theory of killer data acquisitions.

Our framework can be productively extended in many different directions. We have mentioned some of these throughout the article, and due to space constraints, placed the corresponding analysis in the Online Appendix. There are undoubtedly many others that remain to be worked out. The most obvious is to add firm-specific shocks to consumer valuations, while still retaining asymmetric learning functions, which would make the implied industry dynamics more realistic. Our results can then be viewed as what would happen in such a setting in the case that the variance of these stochastic terms becomes very small. Another way to generalize our model is to allow firms to make investments in either the cost of gathering data from their consumers or in the algorithms they use to learn from data, rather than prices being the only strategic choice of the firms. We discussed one way to incorporate depreciation in our setting. Future work could usefully analyze what happens when depreciation is proportional to the total stock of existing data (which is more realistic), possibly using a finite version of our model. Finally, one could also analyze what happens with across-user learning in the case (some) customers have positive mass, which is more natural when customers are businesses rather than regular consumers.

7 Appendix

This appendix contains the proofs of results not already proven in the main text.

7.1 Proof of Proposition 1 and Corollary 1

Given the learning thresholds of the two firms, we can restrict attention without loss of generality to states (N_I, N_E) with $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. As well as establishing $\Delta(N_I, N_E)$ in Proposition 1, we will establish the firms' value functions are given by

$$V^I(N_I, N_E) = \begin{cases} \frac{s_I - s_E}{1-\delta} + \frac{\Delta(N_I,N_E) - \delta\Delta(N_I,N_E+1)}{(1-\delta)^2} & \text{if} \quad s_E - s_I < \Delta(N_I, N_E+1) \\ \frac{s_I - s_E + \Delta(N_I,N_E)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I, N_E+1) \leq s_E - s_I < \Delta(N_I, N_E) \\ 0 & \text{if} \quad s_E - s_I \geq \Delta(N_I, N_E) \end{cases} \tag{9}$$

$$V^E(N_I, N_E) = \begin{cases} 0 & \text{if} \quad s_E - s_I < \Delta(N_I, N_E) \\ \frac{s_E - s_I - \Delta(N_I,N_E)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I+1, N_E) \\ \frac{s_E - s_I}{1-\delta} - \frac{\Delta(N_I,N_E) - \delta\Delta(N_I+1,N_E)}{(1-\delta)^2} & \text{if} \quad s_E - s_I \geq \Delta(N_I+1, N_E) \end{cases}. \tag{10}$$

To establish these expressions, we proceed by two-dimensional induction in (N_I, N_E). We start with $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$, then show the result for all (N_I, N_E) such that $N_I = \overline{N}_I$ (induction in N_E). We can then get the result for all (N_I, N_E) such that $N_E = \overline{N}_E$ by symmetry (induction in N_I). Finally, we show that if the result holds for (N_I+1, N_E) and (N_I, N_E+1), then it also holds for (N_I, N_E), where $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$. Together, these steps allow us to conclude the result holds for any (N_I, N_E) such that $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$.

The state $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$ is already handled in the main text, and the corresponding value function for the firms are given in (1). Consider the case with $N_I = \overline{N}_I$ and $1 \leq N_E \leq \overline{N}_E$. Suppose $\Delta(\overline{N}_I, N_E)$ is given by $\Delta(N_I, N_E)$ in Proposition 1 and (9)-(10) hold with $N_I = \overline{N}_I$ (this is the induction hypothesis).

Now consider the case in which $N_I = \overline{N}_I$ and E has previously sold to $N_E - 1$ consumers. Suppose E charges p^E and I charges p^I in the current period. I wins the current period if $s_I + f_I(\overline{N}_I) - p^I > s_E + f_E(N_E - 1) - p^E$. Otherwise, E wins. If E wins the current period, the PDV of its current and future profits will be $p^E - c + \delta V^E(\overline{N}_I, N_E)$. If it doesn't win the current period, the PDV of its current and future profits will be $\delta V^E(\overline{N}_I, N_E - 1)$. So E is willing to price down to the point where $p^E - c + \delta V^E(\overline{N}_I, N_E) = \delta V^E(\overline{N}_I, N_E - 1)$, or in other words $p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$. Based on the same logic, I is willing to price down to $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$.

Thus, if we denote by $\Omega^i(N_I, N_E)$ the maximum surplus firm i is willing to offer consumers to win in the current period when the current state is (N_I, N_E), then

$$\begin{aligned}
\Omega^I\left(\overline{N}_I, N_E - 1\right) &= s_I - c + f_I(\overline{N}_I) + \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E)) \\
\Omega^E\left(\overline{N}_I, N_E - 1\right) &= s_E - c + f_E(N_E - 1) + \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1)).
\end{aligned}$$

Given Bertrand competition, I wins the current period iff

$$\Omega^I\left(\overline{N}_I, N_E - 1\right) > \Omega^E\left(\overline{N}_I, N_E - 1\right). \tag{11}$$

Next, we determine $V^I(\overline{N}_I, N_E - 1)$ and $V^E(\overline{N}_I, N_E - 1)$ as functions of $\Omega^I\left(\overline{N}_I, N_E - 1\right)$ and $\Omega^E\left(\overline{N}_I, N_E - 1\right)$. Suppose (11) holds. Then in the current period E sets $p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$ and I wins by setting $p^I = s_I + f_I(\overline{N}_I) - \Omega^E\left(\overline{N}_I, N_E - 1\right)$. Then I's value is

$$V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E) + \Omega^I\left(\overline{N}_I, N_E - 1\right) - \Omega^E\left(\overline{N}_I, N_E - 1\right),$$

and E's value is $V^E(\overline{N}_I, N_E - 1) = \delta V^E(\overline{N}_I, N_E - 1)$ given it loses and so will be in the same situation next period. As $\delta < 1$, this implies $V^E(\overline{N}_I, N_E - 1) = 0$.

Conversely, suppose (11) doesn't hold. Then in the current period I sets $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$ and E wins by setting $p^E = s_E + f_E(N_E - 1) - \Omega^I\left(\overline{N}_I, N_E - 1\right)$. Then E's value is

$$V^E(\overline{N}_I, N_E - 1) = \delta V^E(\overline{N}_I, N_E - 1) + \Omega^E\left(\overline{N}_I, N_E - 1\right) - \Omega^I\left(\overline{N}_I, N_E - 1\right)$$

and I's value is $V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E)$ given it loses and will be in the state $(\overline{N}_I, N_E)$ next period.

Combining the above two cases when (11) holds and when it doesn't, we have

$$V^I(\overline{N}_I, N_E - 1) = \delta V^I(\overline{N}_I, N_E) + \max\left\{\Omega^I\left(\overline{N}_I, N_E - 1\right) - \Omega^E\left(\overline{N}_I, N_E - 1\right), 0\right\} \tag{12}$$

$$V^E(\overline{N}_I, N_E - 1) = \delta V^E(\overline{N}_I, N_E - 1) + \max\left\{\Omega^E\left(\overline{N}_I, N_E - 1\right) - \Omega^I\left(\overline{N}_I, N_E - 1\right), 0\right\}. \tag{13}$$

We can now plug the expressions of $\Omega^I\left(\overline{N}_I, N_E - 1\right)$ and $\Omega^E\left(\overline{N}_I, N_E - 1\right)$ above into (12) and (13) to explicitly solve for $V^I(\overline{N}_I, N_E - 1)$ and $V^E(\overline{N}_I, N_E - 1)$. Note that $V^I(\overline{N}_I, N_E - 1)$ will be continuously decreasing in $s_E - s_I$ and $V^E(\overline{N}_I, N_E - 1)$ will be continuously increasing in $s_E - s_I$.

There are two possibilities: $V^E(\overline{N}_I, N_E - 1) = 0$ (i.e. (11) holds and I wins, so even if E were to win the current period, it would still lose in the next period) and $V^E(\overline{N}_I, N_E - 1) > 0$ (i.e. (11) doesn't hold and E wins, so winning the current period would allow E to win in all future periods).

Suppose first $V^E(\overline{N}_I, N_E-1)=0$. We can solve (12) for $V^I(\overline{N}_I, N_E-1)$ using the expressions for $V^I(\overline{N}_I, N_E)$ and $V^E(\overline{N}_I, N_E)$ from the induction hypothesis and the fact that $V^E(\overline{N}_I, N_E-1)=0$, which gives

$$V^I(\overline{N}_I, N_E-1)=\begin{cases} \frac{s_I-s_E+f_I(\overline{N}_I)-f_E(N_E-1)}{1-\delta} & \text{if} \quad s_E-s_I<\Delta(\overline{N}_I, N_E) \\ \frac{s_I-s_E+(1-\delta)(f_I(\overline{N}_I)-f_E(N_E-1))+\delta\Delta(\overline{N}_I, N_E)}{(1-\delta)^2} & \text{if} \quad s_E-s_I\geq\Delta(\overline{N}_I, N_E) \end{cases}.$$

This implies $V^I(\overline{N}_I, N_E-1)\geq 0$ iff

$$s_E-s_I\leq(1-\delta)(f_I(\overline{N}_I)-f_E(N_E-1))+\delta\Delta(\overline{N}_I, N_E)=\Delta(\overline{N}_I, N_E-1).$$

Next, suppose $V^E(\overline{N}_I, N_E-1)>0$, so $V^I(\overline{N}_I, N_E-1)=\delta V^I(\overline{N}_I, N_E)$. As $V^I\left(\overline{N}_I, N_E-1\right)\geq V^I\left(\overline{N}_I, N_E\right)$, we must have $V^I(\overline{N}_I, N_E-1)=V^I(\overline{N}_I, N_E)=0$ in this case. We can solve (13) for $V^E(\overline{N}_I, N_E-1)$ using the expression for $V^E(\overline{N}_I, N_E)$ from the induction hypothesis and the fact that $V^I(\overline{N}_I, N_E-1)=V^I(\overline{N}_I, N_E)=0$, which gives

$$V^E(\overline{N}_I, N_E-1)=\frac{s_E-s_I+(1-\delta)(f_E(N_E-1)-f_I(\overline{N}_I))-\delta\Delta(\overline{N}_I, N_E)}{1-\delta}.$$

This is positive iff $s_E-s_I>\Delta(\overline{N}_I, N_E-1)$.

Thus, combining the results with $V^E(\overline{N}_I, N_E-1)=0$ and $V^E(\overline{N}_I, N_E-1)>0$, we have proven that $\Delta(\overline{N}_I, N_E-1)$, $V^I(\overline{N}_I, N_E-1)$ and $V^E(\overline{N}_I, N_E-1)$ satisfy the characterization in Proposition 1 and (9)-(10), and by induction, this is true for any $0\leq N_E\leq\overline{N}_E$ when $N_I=\overline{N}_I$. Symmetry can be used to obtain a parallel result for all $0\leq N_I\leq\overline{N}_I$ when $N_E=\overline{N}_E$.

Now consider any state (N_I, N_E) with $0\leq N_I\leq\overline{N}_I-1$ and $0\leq N_E\leq\overline{N}_E-1$, and suppose that the results in Proposition 1 and (9)-(10) hold for the states (N_I+1, N_E) and (N_I, N_E+1)—this is the induction hypothesis. Using the same logic as above when the state is (N_I, N_E), we have

$$V^I(N_I, N_E) = \delta V^I(N_I, N_E+1)+\max\{\Omega(N_I, N_E), 0\} \tag{14}$$
$$V^E(N_I, N_E) = \delta V^E(N_I+1, N_E)+\max\{-\Omega(N_I, N_E), 0\} \tag{15}$$

where

$$\begin{aligned}\Omega(N_I, N_E) &= s_I+f_I(N_I)+\delta(V^I(N_I+1, N_E)-V^I(N_I, N_E+1)) \\ &\quad -s_E-f_E(N_E)-\delta\left(V^E(N_I, N_E+1)-V^E(N_I+1, N_E)\right).\end{aligned}$$

Again there are two possibilities, depending on whether $\Omega(N_I, N_E)$ is positive or not. If $\Omega(N_I, N_E)\geq 0$, then we must have $V^E(N_I, N_E)=V^E(N_I+1, N_E)=0$, whereas if $\Omega(N_I, N_E)\leq 0$, then we must have $V^I(N_I, N_E)=V^I(N_I, N_E+1)=0$. When $\Omega(N_I, N_E)\geq 0$ (respectively, $\Omega(N_I, N_E)\leq 0$) we can use the expressions of $V^I(N_I+1, N_E)$ and $V^E(N_I, N_E+1)$ (respectively, $V^I(N_I, N_E+1)$ and $V^E(N_I+1, N_E)$) from the induction hypothesis to solve (14) and

(15) for $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$. After some careful manipulations, which include rewriting the resulting summation expressions for $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ in terms of $\Delta(N_I, N_E)$ and $\Delta(N_I, N_E+1)$, we confirm that the expressions of $\Delta(N_I, N_E)$, $V^I(N_I, N_E)$ and $V^E(N_I, N_E)$ satisfy the characterization in Proposition 1 and (9)-(10) (these additional details are provided in Online Appendix I).

By repeated application of the induction hypothesis, we obtain that (9)-(10), and the expression for $\Delta(N_I, N_E)$ in Proposition 1 hold for all states (N_I, N_E), where $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$.

Finally, to show Proposition 1 continues to apply in the limit as $\overline{N}_I \to \infty$ and $\overline{N}_E \to \infty$, we need to prove the expression in (2) continues to be well defined in this limit case. This is equivalent to proving $\lim_{N_i \to \infty} \sum_{j=0}^{N_i} \delta^j f_i(j)$ exists given this also implies $\lim_{N_i \to \infty} \delta^{N_i} f_i(N_i)$ exists as the summation includes the later expression and other positive terms. Consider the power function $f_i(x) = \theta_i x^{\beta_i}$ where $\theta_i > 0$ and $\beta_i > 0$. Using the ratio test, we know that $\lim_{j\to\infty}(\frac{\delta^{j+1}(j+1)^\beta}{\delta^j j^\beta}) = \delta < 1$ which implies the limit exists for any such power function. Then by the limit comparison test, because the relevant terms are all positive, the limits exist for any unbounded learning function f_i provided there exists some $\kappa_i > 0$, such that $\lim_{j\to\infty} \frac{f_i(j)}{j^\beta} \leq \kappa_i$ for $i = \{I, E\}$.

7.2 Proof of Proposition 2

For (i), suppose first $N_E + k < N_I + k \leq \underline{N}$. Then imposing that learning functions are identical on (3), we get

$$\Delta(N_I + k, N_E + k) - \Delta(N_I, N_E) = \frac{(1-\delta^k)(1-\delta^{\overline{N}-\underline{N}})(\delta^{\overline{N}-N_I} - \delta^{\overline{N}-N_E})\theta}{(1-\delta)\delta^{\overline{N}-\underline{N}+k-1}} > 0.$$

Suppose instead $N_I > N_E > \underline{N}$ and $N_I + k \leq \overline{N}$. Then

$$\Delta(N_I + k, N_E + k) - \Delta(N_I, N_E) = -\frac{(1-\delta^k)(\delta^{\overline{N}-N_I+1} - \delta^{\overline{N}-N_E+1})\theta}{(1-\delta)\delta^k} < 0.$$

For (ii), the new learning curves are

$$\widetilde{f}_i\left(N_i'\right) = f_i\left(N_i\right) + \alpha\left(f_i\left(N_i'\right) - f_i\left(N_i\right)\right)$$

for all $N_i' \geq N_i$. Recall $f_i\left(N_i\right) = \theta_i \max\left\{\min\left\{N_i, \overline{N}_i\right\} - \underline{N}_i, 0\right\}$ for $i = I, E$.

Thus:

- If $N_i \leq \underline{N}_i$ for $i = I, E$, then

$$\widetilde{f}_i\left(N_i'\right) = \alpha f_i\left(N_i'\right) = \alpha\theta_i \max\left\{\min\left\{N_i', \overline{N}_i\right\} - \underline{N}_i, 0\right\}.$$

In this case, we have $\widetilde{\Delta}(N_I, N_E) = \alpha\Delta(N_I, N_E)$, so α amplifies the advantage of the firm that had a competitive advantage to begin with, which is I in this setting.

- If $\underline{N}_i < N_i \leq \overline{N}_i$ for $i = I, E$, then

$$\begin{aligned}\widetilde{f}_i\left(N_i'\right) &= \theta_i\left(N_i - \underline{N}_i\right) + \alpha\theta_i\left(\min\left\{N_i', \overline{N}_i\right\} - N_i\right) \\ &= \alpha\theta_i \min\left\{N_i', \overline{N}_i\right\} - \theta_i\left(\left(\alpha - 1\right)N_i + \underline{N}_i\right).\end{aligned}$$

  Straightforward calculations then lead to the new expression of the threshold

$$\widetilde{\Delta}(N_I, N_E) = \theta_I\left(N_I - \underline{N}_I + \alpha\frac{\delta - \delta^{\overline{N}_I - N_I + 1}}{1-\delta}\right) - \theta_E\left(N_E - \underline{N}_E + \alpha\frac{\delta - \delta^{\overline{N}_E - N_E + 1}}{1-\delta}\right),$$

  which we want to compare to

$$\Delta(N_I, N_E) = \theta_I\left(N_I - \underline{N}_I + \frac{\delta - \delta^{\overline{N}_I - N_I + 1}}{1-\delta}\right) - \theta_E\left(N_E - \underline{N}_E + \frac{\delta - \delta^{\overline{N}_E - N_E + 1}}{1-\delta}\right).$$

  If $\theta_I = \theta_E = \theta$ and $\overline{N}_I = \overline{N}_E = \overline{N}$, then

$$\widetilde{\Delta}(N_I, N_E) - \Delta(N_I, N_E) = (\alpha - 1)\,\theta\frac{\delta^{\overline{N} - N_E + 1} - \delta^{\overline{N} - N_I + 1}}{1-\delta}.$$

  Thus, if $N_I > N_E$, then $\widetilde{\Delta}(N_I, N_E) - \Delta(N_I, N_E) < 0$, so α reduces I's competitive advantage.

7.3 Proof of Proposition 5

The proof follows in three steps. First, we show that $CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right)$, as defined by (4)-(6), is decreasing in $s_E - s_I$ for $s_E - s_I \leq \Delta(N_I, N_E)$ and increasing in $s_E - s_I$ for $s_E - s_I \geq \Delta(N_I, N_E)$. To see this, note

- If $s_E - s_I < \Delta(N_I, N_E + 1)$, then

$$CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right) = \sum_{j=0}^{\infty} \delta^j \min\left\{s_I - s_E + f_I(N_I + j), f_E\left(N_E + j\right)\right\},$$

  which is clearly (weakly) decreasing in $s_E - s_I$.

- If $\Delta(N_I, N_E + 1) \leq s_E - s_I < \Delta(N_I, N_E)$, then

$$CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right) = \sum_{j=0}^{\infty} \delta^j \min\left\{f_I(N_I + j), s_E - s_I + f_E\left(N_E + j\right)\right\} + \frac{s_I - s_E}{(1-\delta)^2} + K,$$

where K is a term constant in $s_E - s_I$. And as

$$\sum_{j=0}^{\infty} \delta^j \left(s_E - s_I\right) = \frac{s_E - s_I}{1-\delta} < \frac{s_E - s_I}{(1-\delta)^2},$$

we conclude that $CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right)$ is strictly decreasing in $s_E - s_I$ on this region.

- The cases with $s_E - s_I \geq \Delta(N_I+1, N_E)$ and $\Delta(N_I, N_E) \leq s_E - s_I < \Delta(N_I+1, N_E)$ are respectively weakly and strictly increasing in $s_E - s_I$, following symmetric arguments to the above two cases.

Second, we show that $CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right) < 0$ when $s_E - s_I = \Delta(N_I, N_E)$. Indeed, in this case we have

$$CS\left(N_I, N_E\right)' \leq \sum_{j=0}^{\infty} \delta^j \left(s_E + f_E(N_E+j)\right) - \frac{c}{1-\delta} = CS\left(N_I, N_E\right).$$

Furthermore, $CS\left(N_I, N_E\right)' = CS\left(N_I, N_E\right)$ is possible in this case iff

$$s_E + f_E\left(N_E + j\right) \leq s_I + f_I(N_I + j)$$

for all $j \geq 0$. However, because

$$s_E - s_I = \Delta(N_I, N_E) = (1-\delta) \sum_{j=0}^{\infty} \delta^j \left(f_I(N_I+j) - f_E(N_E+j)\right),$$

we conclude that $CS\left(N_I, N_E\right)' = CS\left(N_I, N_E\right)$ is only possible in this case if

$$s_E + f_E\left(N_E + j\right) = s_I + f_I(N_I + j)$$

for all $j \geq 0$. Given $\overline{N}_E - N_E > \overline{N}_I - N_I$, this would require $f_E\left(\overline{N}_E\right) = f_E\left(\overline{N}_E - 1\right)$, which is ruled out by assumption. Thus, we must have $CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right) < 0$ when $s_E - s_I = \Delta(N_I, N_E)$.

Third, we show that $CS\left(N_I, N_E\right)' - CS\left(N_I, N_E\right) > 0$ when $s_E - s_I \leq \min\left\{\Delta(N_I, N_E+1), f_I\left(N_I\right) - f_E(N_E)\right\}$ and when $s_E - s_I > \max\left\{\Delta(N_I+1, N_E), f_I\left(N_I\right) - f_E(N_E)\right\}$. In the first case we are in the region where there is no subsidy without data sharing, so we have

$$\begin{aligned} CS\left(N_I, N_E\right)' &= \sum_{j=0}^{\infty} \delta^j \min\left\{s_I + f_I(N_I + j), s_E + f_E\left(N_E + j\right)\right\} - \frac{c}{1-\delta} \\ &> \sum_{j=0}^{\infty} \delta^j \min\left\{s_I + f_I(N_I + j), s_E + f_E\left(N_E\right)\right\} - \frac{c}{1-\delta} \\ &= \sum_{j=0}^{\infty} \delta^j \left(s_E + f_E\left(N_E\right)\right) - \frac{c}{1-\delta} = CS\left(N_I, N_E\right). \end{aligned}$$

And, by a symmetric argument, in the case when $s_E - s_I > \max\left\{\Delta(N_I+1, N_E), f_I\left(N_I\right) - f_E(N_E)\right\}$, we have $CS\left(N_I, N_E\right)' > CS\left(N_I, N_E\right)$.

Together, these three steps, and the continuity of the two functions $CS\left(N_I, N_E\right)$ and $CS\left(N_I, N_E\right)'$ in $s_E - s_I$ imply the result in the proposition.

7.4 Proof of Proposition 6

After the stricter privacy policy is implemented, θ_i becomes $\phi\theta_i$, whereas $\underline{N}_i$ becomes $\frac{N_i}{\phi} - \frac{(1-\phi)N_i}{\phi}$ and $\overline{N}_i$ becomes $\frac{\overline{N}_i}{\phi} - \frac{(1-\phi)N_i}{\phi}$, for $i \in \{I, E\}$. Plugging these values into the expression of the cutoff for S-shaped learning curves provided in (3) with $\underline{N}_i \leq N_i \leq \overline{N}_i$ for $i \in \{I, E\}$, the new cutoff is[21]

$$\begin{aligned} \Delta(N_I, N_E)' &= \theta_I\left(N_I - \underline{N}_I\right) + \phi\theta_I\left(\frac{\delta - \delta^{\frac{\overline{N}_I - N_I}{\phi}+1}}{1-\delta}\right) \\ &\quad - \left(\theta_E\left(N_E - \underline{N}_E\right) + \phi\theta_E\left(\frac{\delta - \delta^{\frac{\overline{N}_E - N_E}{\phi}+1}}{1-\delta}\right)\right). \end{aligned} \tag{16}$$

So the change in the cutoff is

$$\Delta(N_I, N_E)' - \Delta(N_I, N_E) = \frac{\delta}{1-\delta}\left(\theta_I y\left(\overline{N}_I - N_I\right) - \theta_E y\left(\overline{N}_E - N_E\right)\right), \tag{17}$$

where $y\left(m\right) = \delta^m - \phi\delta^{\frac{m}{\phi}} - (1-\phi)$ and $0 < \phi < 1$. The expression in (17) is non-negative when $\theta_E \geq \theta_I$ and $\overline{N}_I - N_I \leq \overline{N}_E - N_E$, and positive when one of the two inequalities is strict. To see this, note $y\left(\overline{N}_E - N_E\right) \leq y\left(\overline{N}_I - N_I\right) < 0$ because $y\left(0\right) = 0$ and $y'\left(m\right) = \left(\delta^m - \delta^{\frac{m}{\phi}}\right)\ln\delta < 0$. This means that the new policy increases I's competitive advantage.

[21] To avoid integer complications, assume there exist integers $m_I \geq 1$ and $m_E \geq 1$ such that $\phi = \frac{\overline{N}_I - N_I}{\overline{N}_I - N_I + m_I} = \frac{\overline{N}_E - N_E}{\overline{N}_E - N_E + m_E}$. Thus, m_I and m_E are the number of additional periods of learning required for each firm to reach its maximum learning threshold due to the stricter privacy policy.

Applying the S-curve to (5), the PDV of consumer surplus when I wins equals

$$CS\left(N_I, N_E\right) = \frac{s_I - c}{1-\delta} + \theta_I \sum_{k=0}^{\overline{N}_I - N_I} \delta^k \left(N_I + k - \underline{N}_I\right) + \frac{\delta^{\overline{N}_I - N_I + 1}}{1-\delta} \theta_I \left(\overline{N}_I - \underline{N}_I\right) - \left(\frac{s_I - s_E + \Delta(N_I, N_E)}{(1-\delta)^2}\right)$$

without the policy. As I wins without the policy, and the policy increases I's competitive advantage, I must continue to win after the policy is introduced. The PDV of consumer surplus with the policy equals

$$CS\left(N_I, N_E\right)' = \frac{s_I - c}{1-\delta} + \theta_I \sum_{k=0}^{\frac{\overline{N}_I - N_I}{\phi}} \delta^k (N_I - \underline{N}_I + \phi k) + \frac{\delta^{\frac{\overline{N}_I - N_I}{\phi}+1}}{1-\delta} \theta_I \left(\overline{N}_I - \underline{N}_I\right) - \left(\frac{s_I - s_E + \Delta(N_I, N_E)'}{(1-\delta)^2}\right),$$

where $\Delta(N_I, N_E)'$ is defined in (16). The change in consumer surplus is

$$\frac{\Delta(N_I, N_E) - \Delta(N_I, N_E)'}{(1-\delta)^2} - \theta_I \sum_{k=0}^{\overline{N}_I - N_I} k\delta^k \left(1 - \phi\right) \tag{18}$$

$$-\frac{\theta_I \left(\delta^{\overline{N}_I - N_I + 1} - \delta^{\frac{\overline{N}_I - N_I}{\phi}+1}\right)}{1-\delta} \left(\overline{N}_I - \underline{N}_I\right) + \theta_I \sum_{k=\overline{N}_I - N_I + 1}^{\frac{\overline{N}_I - N_I}{\phi}} \delta^k (N_I - \underline{N}_I + \phi k). \tag{19}$$

Given $\Delta(N_I, N_E)' > \Delta(N_I, N_E)$, the two terms in (18) are negative. Using $\phi = \frac{\overline{N}_I - N_I}{\overline{N}_I - N_I + m_I}$, the expression in (19) can be rewritten as

$$-\delta^{\overline{N}_I - N_I + 1} \frac{\overline{N}_I - N_I}{(1-\delta)^2 \left(\overline{N}_I - N_I + m_I\right)} \left(m_I \left(1-\delta\right) - \left(1 - \delta^{m_I}\right)\right)$$

which is equal to zero if $m_I = 1$, and is negative for $m_I > 1$ given $m_I \left(1-\delta\right) - \left(1-\delta^{m_I}\right) = \left(1-\delta\right)\left(m_I - \sum_{k=1}^{m_I} \delta^{k-1}\right)$.

7.5 Proof of Proposition 7

First, if $s_E - s_I < \Delta\left(N_I, N_E + N_A\right)$, then E is not willing to pay anything for N_A because even if it acquires it, it's not enough to make it win. This also means that E can't have been winning originally without the data, so it makes zero profits regardless of who acquires N_A. In this case, I wins the data as it is willing to pay up to the value it would get if it wins the data less the value it would get if E wins the data—and it is straightforward to verify that this difference is positive if $N_I < \overline{N}_I$ or $N_E < \overline{N}_E$.

Second, if $s_E - s_I \geq \Delta\left(N_I + N_A, N_E\right)$, then we are in the symmetric case where I is willing to pay nothing for the data and E acquires it.

Suppose now we are in the remaining (and interesting) case

$$\Delta(N_I, N_E + N_A) \le s_E - s_I < \Delta(N_I + N_A, N_E),$$

which means that whichever firm acquires the data will win in all periods. Note that we also have

$$\Delta(N_I, N_E + N_A) \le \Delta(N_I + 1, N_E + N_A) \le \Delta(N_I + N_A, N_E),$$

with strict inequalities if $N_I < \overline{N}_I$ and $N_E < \overline{N}_E$. Consequently, using (9)-(10), E's willingness-to-pay for the data is $W_E(s_E - s_I)$, where

$$W_E(x) = \begin{cases} \frac{x - \Delta(N_I, N_E + N_A)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I, N_E + N_A) \le x < \Delta(N_I + 1, N_E + N_A) \\ \frac{x}{1-\delta} - \frac{\Delta(N_I, N_E+N_A) - \delta\Delta(N_I+1, N_E+N_A)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I + 1, N_E + N_A) \le x < \Delta(N_I + N_A, N_E) \end{cases}.$$

The two cases in this expression correspond to whether I subsidizes after E wins the data or not.

Similarly, I's willingness to pay for the data is $W_I(s_E - s_I)$, where

$$W_I(x) = \begin{cases} \frac{-x}{1-\delta} + \frac{\Delta(N_I+N_A, N_E) - \delta\Delta(N_I+N_A, N_E+1)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I, N_E + N_A) \le x < \Delta(N_I + N_A, N_E + 1) \\ \frac{-x + \Delta(N_I+N_A, N_E)}{(1-\delta)^2} & \text{if} \quad \Delta(N_I + N_A, N_E + 1) \le x < \Delta(N_I + N_A, N_E) \end{cases}$$

Note that $W_E(x)$ is increasing in x and $W_I(x)$ is decreasing in x. Furthermore, $W_E(\Delta(N_I, N_E + N_A)) = 0$ and $\lim_{x \to \Delta(N_I+N_A, N_E)} W_I(x) = 0$. Thus, as $W_E(x)$ and $W_I(x)$ are continuous, $\Delta(N_I, N_E + N_A) < \Delta(N_I + N_A, N_E)$, and combining with the two other cases discussed above, we conclude that there is a unique Δ^* such that I's willingness-to-pay for the data is larger (and therefore I wins) when $s_E - s_I < \Delta^*$ and E's willingness-to-pay is larger (and therefore E wins) for $\Delta^* \ge s_E - s_I$. Given we assumed we are in the situation where at least one firm is not already at its learning threshold, the cutoff Δ^* satisfies (7). Noting that $\Delta(N_I + 1, N_E + N_A) \le \Delta(N_I + N_A, N_E + 1)$ and that the socially efficient condition for E to win the data is $s_E - s_I \ge \Delta(N_I + N_A, N_E + N_A)$, we obtain the result stated in the Proposition.[22]

7.6 Proof of Proposition 10

As discussed in the text, the analysis can be conducted with a representative consumer. We will show that the firms' value functions are

$$V^I(N_I, N_E) = \max\left\{\frac{s_I - s_E + \Delta(N_I, N_E)}{1-\delta}, 0\right\} \tag{20}$$

$$V^E(N_I, N_E) = \max\left\{\frac{s_E - s_I - \Delta(N_I, N_E)}{1-\delta}, 0\right\}, \tag{21}$$

[22]The explicit characterization of Δ^* is relegated to Online Appendix C.

where $\Delta(N_I, N_E)$ is given by (2) in the main text.

A key difference with the proof of Proposition 1 is that here we need to construct the value function for the consumer as the consumer needs to take future utility into account when deciding which firm to buy from. We denote this $u(N_I, N_E)$, which measures the PDV of surplus a consumer expects to obtain from making her optimal choices in every stage when firms play their equilibrium strategies and the consumer has previously bought N_I times from firm I and N_E times from firm E.

As in the case with across user learning, the state (N_I, N_E) is equivalent to the state $\left(\overline{N}_I, N_E\right)$ for all $N_I \geq \overline{N}_I$. And similarly, the state (N_I, N_E) is equivalent to the state $\left(N_I, \overline{N}_E\right)$ for all $N_E \geq \overline{N}_E$. So we can restrict attention to states (N_I, N_E) with $0 \leq N_I \leq \overline{N}_I$ and $0 \leq N_E \leq \overline{N}_E$. It is easily verified that when $(N_I, N_E) = (\overline{N}_I, \overline{N}_E)$, we have

$$
\begin{aligned}
V^I(\overline{N}_I, \overline{N}_E) &= \frac{\max\left\{s_I + f_I(\overline{N}_I) - s_E - f_E(\overline{N}_E), 0\right\}}{1-\delta} \\
V^E(\overline{N}_I, \overline{N}_E) &= \frac{\max\left\{s_E + f_E(\overline{N}_E) - s_I - f_I(\overline{N}_I), 0\right\}}{1-\delta} \\
u(\overline{N}_I, \overline{N}_E) &= \frac{\min\left\{s_I + f_I(\overline{N}_I), s_E + f_E(\overline{N}_E)\right\} - c}{1-\delta}.
\end{aligned}
$$

Consider the case with $N_I = \overline{N}_I$ and $1 \leq N_E \leq \overline{N}_E$. Suppose that $\Delta(\overline{N}_I, N_E)$, $V^I(\overline{N}_I, N_E)$ and $V^E(\overline{N}_I, N_E)$ are given by (2), (20) and (21) with $N_I = \overline{N}_I$, and that the corresponding value function for consumers is

$$
u(\overline{N}_I, N_E) = \min\left\{\frac{s_I + f_I(\overline{N}_I) - c}{1-\delta}, \sum_{j=0}^{\infty} \delta^j f_E(N_E + j) + \frac{s_E - c}{1-\delta}\right\}.
$$

This is the induction hypothesis.

Consider now the state $(\overline{N}_I, N_E - 1)$. The consumer chooses I in the current period iff

$$
s_I + f_I(\overline{N}_I) - p^I + \delta u(\overline{N}_I, N_E - 1) > s_E + f_E(N_E - 1) - p^E + \delta u(\overline{N}_I, N_E).
$$

I is willing to set p^I down to the point where it is indifferent between winning or losing the current period, i.e. $p^I = c - \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E))$. Similarly, E is willing to set p^E down to $p^E = c - \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1))$.

If we denote by $\Omega^i(N_I, N_E)$ the PDV of the maximum surplus firm i is willing to offer consumers to win in the current period when the current state is (N_I, N_E), then

$$
\begin{aligned}
\Omega^I\left(\overline{N}_I, N_E - 1\right) &= s_I - c + f_I(\overline{N}_I) + \delta(V^I(\overline{N}_I, N_E - 1) - V^I(\overline{N}_I, N_E) + u(\overline{N}_I, N_E - 1)) \\
\Omega^E\left(\overline{N}_I, N_E - 1\right) &= s_E - c + f_E(N_E - 1) + \delta(V^E(\overline{N}_I, N_E) - V^E(\overline{N}_I, N_E - 1) + u(\overline{N}_I, N_E)).
\end{aligned}
$$

Given Bertrand competition, I wins the current period iff

$$\Omega^I\left(\overline{N}_I, N_E-1\right) > \Omega^E\left(\overline{N}_I, N_E-1\right).$$

Using the same logic as in the Proof of Proposition 1, we obtain

$$\begin{aligned}
u(\overline{N}_I, N_E-1) &= \min\left\{\Omega^I\left(\overline{N}_I, N_E-1\right), \Omega^E\left(\overline{N}_I, N_E-1\right)\right\} && (22)\\
V^I(\overline{N}_I, N_E-1) &= \delta V^I(\overline{N}_I, N_E) + \max\left\{\Omega^I\left(\overline{N}_I, N_E-1\right) - \Omega^E\left(\overline{N}_I, N_E-1\right), 0\right\} && (23)\\
V^E(\overline{N}_I, N_E-1) &= \delta V^E(\overline{N}_I, N_E-1) + \max\left\{\Omega^E\left(\overline{N}_I, N_E-1\right) - \Omega^I\left(\overline{N}_I, N_E-1\right), 0\right\}. && (24)
\end{aligned}$$

There are two possibilities: $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$ or $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$.

Suppose first $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$, which immediately implies $V^E(\overline{N}_I, N_E-1) = 0$. Using the expressions of $V^I(\overline{N}_I, N_E)$, $V^E(\overline{N}_I, N_E)$ and $u(\overline{N}_I, N_E)$ from the induction hypothesis, we then solve (22) and (23) for $V^I(\overline{N}_I, N_E-1)$ and $u(\overline{N}_I, N_E-1)$ to obtain

$$\begin{aligned}
V^I(\overline{N}_I, N_E-1) &= \frac{s_I - s_E + \Delta(\overline{N}_I, N_E-1)}{1-\delta}\\
u(\overline{N}_I, N_E-1) &= \frac{s_E - c}{1-\delta} + \sum_{j=0}^{\infty}\delta^j f_E(N_E-1+j). && (25)
\end{aligned}$$

Now suppose $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$, which means E wins the current period, so $V^I(\overline{N}_I, N_E-1) = \delta V^I(\overline{N}_I, N_E)$, which implies $V^I(\overline{N}_I, N_E-1) = V^I(\overline{N}_I, N_E) = 0$ because we must have $V^I(\overline{N}_I, N_E-1) \geq V^I(\overline{N}_I, N_E)$. Furthermore, (22) and (24) become

$$\begin{aligned}
u(\overline{N}_I, N_E-1) &= \frac{s_I + f_I(\overline{N}_I) - c}{1-\delta} && (26)\\
V^E(\overline{N}_I, N_E-1) &= \frac{s_E - s_I - \Delta(\overline{N}_I, N_E-1)}{1-\delta}.
\end{aligned}$$

Thus, combining the results with $\Omega^I\left(\overline{N}_I, N_E-1\right) \geq \Omega^E\left(\overline{N}_I, N_E-1\right)$ and $\Omega^I\left(\overline{N}_I, N_E-1\right) < \Omega^E\left(\overline{N}_I, N_E-1\right)$, we have proven that $\Delta(\overline{N}_I, N_E-1)$, $V^I(\overline{N}_I, N_E-1)$ and $V^E(\overline{N}_I, N_E-1)$ satisfy the characterization in (2), (20) and (21), and $u(\overline{N}_I, N_E-1)$ is given by the minimum of the two expressions in (25) and (26). By induction, this is true for any $0 \leq N_E \leq \overline{N}_E$ when $N_I = \overline{N}_I$. Symmetry can then be used to obtain a parallel result for all $0 \leq N_I \leq \overline{N}_I$ when $N_E = \overline{N}_E$.

Now consider any state (N_I, N_E) with $0 \leq N_I \leq \overline{N}_I - 1$ and $0 \leq N_E \leq \overline{N}_E - 1$. The induction hypothesis is that the expressions (2), (20) and (21) hold for the states (N_I+1, N_E) and (N_I, N_E+1) and that the value functions for consumers in these two states are

$$u(N_I+1, N_E) = \min\left\{\sum_{j=0}^{\infty}\delta^j f_I(N_I+1+j) + \frac{s_I - c}{1-\delta}, \sum_{j=0}^{\infty}\delta^j f_E(N_E+j) + \frac{s_E - c}{1-\delta}\right\}$$

$$u(N_I,N_E+1)=\min\left\{\sum_{j=0}^{\infty}\delta^j f_I(N_I+j)+\frac{s_I-c}{1-\delta},\sum_{j=0}^{\infty}\delta^j f_E(N_E+1+j)+\frac{s_E-c}{1-\delta}\right\}.$$

Using the same logic as above, we have

$$u(N_I,N_E)=\min\left\{\Omega^I(N_I,N_E),\Omega^E(N_I,N_E)\right\}, \tag{27}$$

where

$$\begin{aligned}\Omega^I(N_I,N_E) &= s_I-c+f_I(N_I)+\delta(V^I(N_I+1,N_E)-V^I(N_I,N_E+1)+u(N_I+1,N_E))\\ \Omega^E(N_I,N_E) &= s_E-c+f_E(N_E)+\delta(V^E(N_I,N_E+1)-V^E(N_I+1,N_E)+u(N_I,N_E+1))\end{aligned}$$

and

$$V^I(N_I,N_E) = \delta V^I(N_I,N_E+1)+\max\left\{\Omega(N_I,N_E),0\right\} \tag{28}$$

$$V^E(N_I,N_E) = \delta V^E(N_I+1,N_E)+\max\left\{-\Omega(N_I,N_E),0\right\}, \tag{29}$$

where $\Omega(N_I,N_E)\equiv\Omega^I(N_I,N_E)-\Omega^E(N_I,N_E)$.

Again there are two possibilities, depending on whether $\Omega(N_I,N_E)>0$ or not. If $\Omega(N_I,N_E)>0$, then we must have $V^E(N_I,N_E)=V^E(N_I+1,N_E)=0$, whereas if $\Omega(N_I,N_E)\leq 0$, then we must have $V^I(N_I,N_E)=V^I(N_I,N_E+1)=0$. For each of these cases, we can use the expressions of $u(N_I+1,N_E)$, $u(N_I,N_E+1)$, $V^I(N_I,N_E+1)$ and $V^E(N_I+1,N_E)$ from the induction hypothesis to solve (27), (28) and (29) for $V^I(N_I,N_E)$, $V^E(N_I,N_E)$ and $u\left(N_I,N_E\right)$. Then we can rewrite the resulting summation expressions in $V^I(N_I,N_E)$ and $V^E(N_I,N_E)$ in terms of $\Delta(N_I,N_E)$ to obtain

$$\begin{aligned}V^I(N_I,N_E) &= \max\left\{\frac{s_I-s_E+\Delta(N_I,N_E)}{1-\delta},0\right\}\\ V^E(N_I,N_E) &= \max\left\{\frac{s_E-s_I-\Delta(N_I,N_E)}{1-\delta},0\right\}\end{aligned}$$

and

$$u(N_I,N_E)=\min\left\{\sum_{j=0}^{\infty}\delta^j f_I(N_I+j)+\frac{s_I-c}{1-\delta},\sum_{j=0}^{\infty}\delta^j f_E(N_E+j)+\frac{s_E-c}{1-\delta}\right\}.$$

By repeated application of the induction hypothesis, the results in Proposition 10 (as well as the expressions (20) and (21)) hold for all states (N_I,N_E), with $0\leq N_I\leq\overline{N}_I$ and $0\leq N_E\leq\overline{N}_E$.

Finally, we can confirm that consumers do not want to switch firms along the equilibrium path. Suppose the current state is (N_I,N_E). If $s_E-s_I<\Delta(N_I,N_E)$, then consumers choose I in the current period, so next period the state will be (N_I+1,N_E). Given $\Delta(N_I,N_E)$ is increasing in N_I, this means in the subsequent period consumers will choose I again, and so on until $N_I=\overline{N}_I$. When the state is $(\overline{N}_I,N_E)$, all subsequent periods will have an identical state and outcome, in which consumers choose I. Conversely, if $s_E - s_I \geq \Delta(N_I, N_E)$, then consumers choose E in the current period, so next period the state will be $(N_I, N_E + 1)$. Given $\Delta(N_I, N_E)$ is decreasing in N_E, this means in the subsequent period consumers will choose E again, and so on until $N_E = \overline{N}_E$. At the state $(N_I, \overline{N}_E)$, all subsequent periods will have an identical state and outcome, in which consumers choose E.

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