

Online Appendix

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A Uncertainty about second-period values

We extend our baseline model with forward-looking consumers by assuming there is uncertainty about the two firms' net values in the second period (with data). Specifically, suppose that from the perspective of period 1, v'_A is distributed with c.d.f. $F_A(\cdot)$ and v'_B is distributed with c.d.f. $F_B(\cdot)$. Their realizations only become known in period 2.

Start with the case without data portability. Following the analysis from the baseline, firm A wins in period 1 iff

$$v_A - p_A + \delta E[u(A)] \geq v_B - p_B + \delta E[u(B)],$$

where

$$\begin{aligned} E[u(A)] &= E[\min\{v'_A, v_B\}] \\ E[u(B)] &= E[\min\{v_A, v'_B\}]. \end{aligned}$$

The maximum subsidy (i.e., price below cost) that firm i is willing to offer the consumer in period 1 in order to win is

$$\begin{aligned} p_i - c &= \delta (E[\pi_i(j)] - E[\pi_i(i)]) \\ &= \delta (E[\max\{v_i - v'_j, 0\}] - E[\max\{v'_i - v_j, 0\}]), \end{aligned}$$

where $i \neq j \in \{A, B\}$.

Thus, A wins in period 1 iff

$$v_A + \delta E[S(A)] \geq v_B + \delta E[S(B)],$$

where recall

$$\begin{aligned} S(A) &= \pi_A(A) + \pi_B(A) + u(A) = \max\{v'_A, v_B\} \\ S(B) &= \max\{v_A, v'_B\}. \end{aligned}$$

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Thus, A's total profits are

$$\max \{v_A - v_B + \delta E [S(A) - S(B)], 0\} + \delta E [\pi_A(B)].$$

By the same logic, A's profits when it offers data portability and B does not are

$$\max \{v_A - v_B + \delta E [S^P - S(B)], 0\} + \delta E [\pi_A(B)].$$

Similarly, A's total profits when it does not offer data portability and B does is

$$\max \{v_A - v_B + \delta E [S(A) - S^P], 0\} + \delta E [\pi_A^P].$$

And A's profits when it offers data portability and B does are

$$\max \{v_A - v_B + \delta E [S^P - S^P], 0\} + \delta E [\pi_A^P].$$

Thus, regardless of whether B offers data portability or not, A prefers to commit to data portability iff

$$E [S^P] \geq E [S(A)],$$

which is equivalent to

$$E [\max \{v'_A, v'_B\}] \geq E [\max \{v'_A, v_B\}].$$

Similarly, B prefers to commit to data portability iff

$$E [\max \{v'_A, v'_B\}] \geq E [\max \{v_A, v'_B\}].$$

Note that, under independence of the draws v'_A and v'_B , simple sufficient conditions for these inequalities to hold are $E [v'_B] \geq v_B$ and $E [v'_A] \geq v_A$.

B Non-negative price constraint

In this section, we assume $\min\{v_A, v_B\} \geq 0$ and impose a non-negative price constraint on both firms. First, we show that each firm still weakly prefer to commit to data portability regardless of what the other firm does, and strictly so when the advantage switches with data and an additional condition is satisfied. Second, we also show that now bilateral data portability can increase consumer surplus (unlike in the baseline).

B.1 Equilibrium analysis

Suppose first $v'_A > v_A > v'_B > v_B$. The price constraint does not bind because firm A always wins both periods and firm B is never willing to subsidize. Thus, nothing changes in this case relative to the baseline analysis. Both A and B are still indifferent about offering data portability. By symmetry, the same is true when $v'_B > v_B > v'_A > v_A$.

Next, suppose $v'_A > v'_B > v_A > v_B$. Without data portability, the unconstrained minimum period-1 prices the two firms are willing to offer are

$$\begin{aligned} p_A^m &= c + \delta(\pi_A(B) - \pi_A(A)) = c - \delta(v'_A - v_B) \\ p_B^m &= c + \delta(\pi_B(A) - \pi_B(B)) = c - \delta(v'_B - v_A). \end{aligned}$$

Since $v'_A - v_B > v'_B - v_A$, there are only two cases in which the constraint binds. Either (i) $\delta(v'_A - v_B) > c > \delta(v'_B - v_A)$, so $p_A^m = 0$ and $p_B^m = c - \delta(v'_B - v_A) > 0$ or (ii) $\delta(v'_A - v_B) > \delta(v'_B - v_A) \geq c$, so $p_A^m = p_B^m = 0$. Combining the two cases, if the constraint binds, then

$$\begin{aligned} p_A^m &= 0 \\ p_B^m &= \max\{c - \delta(v'_B - v_A), 0\}. \end{aligned}$$

Recall that firm A wins iff

$$v_A - p_A^m + \delta u(A) \geq v_B - p_B^m + \delta u(B),$$

which, when the constraint binds, is equivalent to

$$(1 - \delta)(v_A - v_B) + p_B^m \geq 0.$$

This is always true because $v_A > v_B$ and $p_B^m \geq 0$. Thus, A will always win. The period-1 prices are then

$$\begin{aligned} p_B &= p_B^m \\ p_A &= v_A - v_B + p_B^m + \delta(u(A) - u(B)) = (1 - \delta)(v_A - v_B) + p_B^m > 0, \end{aligned}$$

so A's discounted sum of profits across the two periods is

$$\begin{aligned} \Pi_A &= p_A - c + \delta\pi_A(A) \\ &= (1 - \delta)(v_A - v_B) + p_B^m - c + \delta(v'_A - v_B) \\ &= (1 - \delta)(v_A - v_B) + \max\{c - \delta(v'_B - v_A), 0\} - c + \delta(v'_A - v_B). \end{aligned}$$

Firm B makes zero profits.

Now suppose A offers data portability. The unconstrained minimum period-1 prices the two firms are willing to offer are

$$\begin{aligned} p_A^m &= c + \delta (\pi_A(B) - \pi_A^P) = c - \delta (v'_A - v'_B) \\ p_B^m &= c + \delta (\pi_B^P - \pi_B(B)) = c - \delta (v'_B - v_A). \end{aligned}$$

Recall case (i) arises when $\delta (v'_A - v_B) > c > \delta (v'_B - v_A)$. In this case, $p_A^m = \max \{c - \delta (v'_A - v'_B), 0\}$ and $p_B^m = c - \delta (v'_B - v_A) > 0$, while in case (ii) we have $\delta (v'_B - v_A) \geq c$, so $p_A^m = \max \{c - \delta (v'_A - v'_B), 0\}$ and $p_B^m = 0$. Firm A always wins period 2 regardless of who wins in period 1, because $v'_A > v'_B$. And firm A also wins period 1 iff

$$v_A - p_A^m + \delta u^P \geq v_B - p_B^m + \delta u(B).$$

In case (i), this becomes

$$v_A - \max \{c - \delta (v'_A - v'_B), 0\} + \delta v'_B \geq v_B - (c - \delta (v'_B - v_A)) + \delta v_A.$$

Even if $c - \delta (v'_A - v'_B) \geq 0$, this inequality holds, so A always wins in case (i).

In case (ii), firm A wins period 1 iff

$$v_A - \max \{c - \delta (v'_A - v'_B), 0\} + \delta v'_B \geq v_B + \delta v_A.$$

Even if $c - \delta (v'_A - v'_B) \geq 0$, this inequality holds (because the inequality for case (ii) $\delta (v'_B - v_A) \geq c$ implies $\delta (v'_A - v_A) \geq c$), so again A always wins in case (ii).

In case (i), the first-period prices are

$$\begin{aligned} p_B &= p_B^m = c - \delta (v'_B - v_A) \\ p_A &= v_A - v_B + p_B^m + \delta (u^P - u(B)) = v_A - v_B + c - \delta (v'_B - v_A) + \delta (v'_B - v_A) = c + v_A - v_B > 0, \end{aligned}$$

so A's discounted sum of profits across the two periods is

$$\Pi_A^P = p_A - c + \delta \pi_A^P = v_A - v_B + \delta (v'_A - v'_B),$$

which is identical to its profit in the case without data portability. Thus, firm A is indifferent about adopting data portability.

In case (ii), the first-period prices are

$$\begin{aligned} p_B &= p_B^m = 0 \\ p_A &= v_A - v_B + p_B^m + \delta(u^P - u(B)) = v_A - v_B + \delta(v'_B - v_A), \end{aligned}$$

so

$$\begin{aligned} \Pi_A^P &= p_A - c + \delta\pi_A^P \\ &= v_A - v_B + \delta(v'_B - v_A) - c + \delta(v'_A - v'_B) \\ &= v_A - v_B + \delta(v'_A - v_A) - c, \end{aligned}$$

which is also identical to the case without data portability. Firm B's profits are also identical and equal to zero.

Thus, firm A is indifferent about adopting data portability in this case. A similar analysis establishes that A is also indifferent about adopting data portability if B already offers it. And B is indifferent about adopting data portability regardless of what A does, because B earns zero profits in all scenarios. By symmetry in A and B, the same conclusion holds when $v'_B > v'_A > v_B > v_A$.

Next, suppose $v'_B > v'_A > v_A > v_B$. Without data portability, the unconstrained minimum period-1 prices the two firms are willing to offer are

$$\begin{aligned} p_A^m &= c + \delta(\pi_A(B) - \pi_A(A)) = c - \delta(v'_A - v_B) \\ p_B^m &= c + \delta(\pi_B(A) - \pi_B(B)) = c - \delta(v'_B - v_A). \end{aligned}$$

There are three cases where the non-negative price constraint binds: (i) $\delta(v'_A - v_B) > c > \delta(v'_B - v_A)$, so $p_A^m = 0$ and $p_B^m = c - \delta(v'_B - v_A) > 0$; (ii) $\delta(v'_A - v_B) \geq c$ and $\delta(v'_B - v_A) \geq c$, so $p_A^m = p_B^m = 0$; and (iii) $\delta(v'_B - v_A) > c > \delta(v'_A - v_B)$, so $p_A^m = c - \delta(v'_A - v_B) > 0$ and $p_B^m = 0$.

Cases (i) and (ii) lead to the same analysis as before, so A always wins in both cases and its profits are

$$\Pi_A = (1 - \delta)(v_A - v_B) + \max\{c - \delta(v'_B - v_A), 0\} - c + \delta(v'_A - v_B).$$

Now consider case (iii). Firm A wins iff

$$v_A - p_A^m + \delta u(A) \geq v_B - p_B^m + \delta u(B),$$

which is equivalent to

$$(1 - \delta)v_A + \delta v'_A \geq c + v_B.$$

If $(1 - \delta)v_A + \delta v'_A \geq c + v_B$ then A wins, $p_B = 0$, $p_A = (1 - \delta)(v_A - v_B)$, and

$$\begin{aligned}\Pi_A &= p_A - c + \delta\pi_A(A) = (1 - \delta)(v_A - v_B) - c + \delta(v'_A - v_B) \\ &= (1 - \delta)v_A - v_B - c + \delta v'_A.\end{aligned}$$

Otherwise, A's profit in both periods is zero. Combining the two cases, A's discounted sum of profits across the two periods is

$$\Pi_A = \max\{(1 - \delta)v_A - v_B - c + \delta v'_A, 0\}.$$

Now suppose A offers data portability. The unconstrained minimum period-1 prices the two firms are willing to offer are

$$\begin{aligned}p_A^m &= c + \delta(\pi_A(B) - \pi_A^P) = c > 0 \\ p_B^m &= c + \delta(\pi_B^P - \pi_B(B)) = c + \delta(v'_B - v'_A - (v'_B - v_A)) = c - \delta(v'_A - v_A).\end{aligned}$$

Recall the three cases above in which the non-negative price constraint can bind:

- (i) $\delta(v'_A - v_B) > c > \delta(v'_B - v_A)$, so $p_B^m = c - \delta(v'_A - v_A) > 0$;
- (ii) $\delta(v'_A - v_B) \geq c$ and $\delta(v'_B - v_A) \geq c$, so $p_B^m = c - \delta(v'_A - v_A)$ is either positive or negative; and
- (iii) $\delta(v'_B - v_A) > c > \delta(v'_A - v_B)$, so $p_B^m = c - \delta(v'_A - v_A) > 0$.

Firm A wins period 1 iff $v_A - p_A^m + \delta u^P \geq v_B - \max\{p_B^m, 0\} + \delta u(B)$. If $p_B^m = c - \delta(v'_A - v_A) > 0$, this becomes

$$v_A - c + \delta v'_A \geq v_B - c + \delta(v'_A - v_A) + \delta v_A,$$

which is clearly true because $v_A > v_B$. And if $p_B^m = c - \delta(v'_A - v_A) \leq 0$, the condition for A to win becomes

$$v_A - c + \delta v'_A \geq v_B + \delta v_A,$$

which also holds because in this case $c \leq \delta(v'_A - v_A)$.

Thus, in all cases, A wins in period 1, consistent with the baseline analysis without the non-negative price constraint. And B always wins in period 2 due to data portability and $v'_B > v'_A$ in this region. So A's profits are

$$\Pi_A^{PA} = p_A - c = (1 - \delta)v_A - v_B - c + \delta v'_A + \max\{c - \delta(v'_A - v_A), 0\},$$

which are positive and clearly weakly higher than profits without data portability,

$$\Pi_A = \max \{(1 - \delta) v_A - v_B - c + \delta v'_A, 0\}.$$

A is indifferent if and only if $c \leq \delta (v'_A - v_A)$, otherwise A's preference for data portability is strict.

Meanwhile, B's profits when only A offers data portability are simply

$$\Pi_B^{PA} = \delta (v'_B - v'_A).$$

If both firms offer data portability, then neither firm is willing to subsidize in period 1, A wins period 1 and B wins period 2, leading to profits

$$\begin{aligned} \Pi_A^P &= v_A - v_B \\ \Pi_B^P &= \delta (v'_B - v'_A). \end{aligned}$$

This means that starting from a situation in which only A offers data portability, B is indifferent between offering it and not.

Finally, suppose only B has committed to data portability. The unconstrained minimum period-1 prices the two firms are willing to offer are

$$\begin{aligned} p_A^m &= c + \delta (\pi_A^P - \pi_A(A)) = c - \delta (v'_A - v_B) \\ p_B^m &= c + \delta (\pi_B(A) - \pi_B^P) = c - \delta (v'_B - v'_A). \end{aligned}$$

There are three cases where the non-negative price constraint binds: (i) $\delta (v'_A - v_B) > c > \delta (v'_B - v'_A)$, so $p_A^m = 0$ and $p_B^m = c - \delta (v'_B - v'_A) > 0$; (ii) $\delta (v'_A - v_B) \geq c$ and $\delta (v'_B - v'_A) \geq c$, so $p_A^m = p_B^m = 0$; and (iii) $\delta (v'_B - v'_A) > c > \delta (v'_A - v_B)$, so $p_A^m = c - \delta (v'_A - v_B) > 0$ and $p_B^m = 0$.

Firm A wins both periods iff

$$v_A - p_A^m + \delta u(A) \geq v_B - p_B^m + \delta u^P.$$

In case (i), this is equivalent to

$$v_A - v_B - \delta (v'_B - v_B) + c \geq 0.$$

If this holds, A's profits are

$$\Pi_A^{PB} = v_A - v_B - \delta (v'_B - v'_A).$$

Otherwise, A makes zero profits in case (i). In case (ii), the condition for A to win becomes

$$v_A + \delta v_B \geq v_B + \delta v'_A.$$

If it holds, then A's profits are

$$\Pi_A^{PB} = v_A - v_B - c.$$

Otherwise, A makes zero profits in case (ii). In case (iii), the condition for A to win becomes

$$v_A - v_B \geq c.$$

If it holds, then A's profits are

$$\Pi_A^{PB} = v_A - v_B - c.$$

Otherwise, A makes zero profits in case (iii).

Now suppose A offers data portability as well, in which case we already know that A's profits are simply

$$\Pi_A^P = v_A - v_B.$$

It follows that $\Pi_A^P > \Pi_A^{PB}$ in all cases provided $c > 0$.

In summary, A weakly prefers to offer data portability regardless of what B does and vice versa for B, with strict preference for some consumer configurations. This implies it is an equilibrium for both firms offer data portability. By symmetry in A and B, the same conclusion holds when $v'_A > v'_B > v_B > v_A$.

In conclusion, as in the baseline analysis, each firm weakly prefers to commit to data portability regardless of what the other firm does. The difference is that the preference is strict on a smaller range of consumer configurations than in the baseline without a non-negative price constraint.

B.2 Consumers could be better off

We now compare consumer surplus when data portability is ruled out with consumer surplus when both firms adopt data portability. In the baseline analysis without the non-negative price constraint, bilateral data portability makes consumers weakly worse off (sometimes strictly). With the non-negative price constraint, this conclusion no longer necessarily holds. The reason is that without data portability, the firm with the weaker initial product may be unable to offer a sufficiently low first-period price which passes on to the consumer the value it could generate in the future. In this case, data portability might make consumers better off.

To see this most simply, suppose

$$v'_A > v'_B > v_A > v_B.$$

In this case, A has the advantage both initially and with data. If both firms offer data portability, A wins in period 1 and also wins in period 2, but B has access to the consumer's data in period 2. In period 2, A therefore wins and the consumer obtains the net value offered by B, namely v'_B . Consumer surplus under bilateral data portability is

$$CS^P = v_B + \delta v'_B.$$

Now suppose data portability is ruled out. If A wins in period 1, then A wins in period 2 and the consumer obtains $u(A) = v_B$ in period 2. If B wins in period 1, then B wins in period 2 and the consumer obtains $u(B) = v_A$ in period 2. The incremental period-2 profit from winning rather than losing for B is therefore $v'_B - v_A$. Hence B's constrained minimum price in period 1 is

$$p_B^m = \max \{0, c - \delta (v'_B - v_A)\}.$$

At this price, the maximum utility B can offer the consumer is

$$v_B + c - p_B^m + \delta v_A = v_B + \delta v_A + \min \{c, \delta (v'_B - v_A)\}.$$

Since A wins both periods, it sets its period-1 price so that the consumer is just willing to choose A rather than B. Thus consumer surplus without data portability is

$$CS = v_B + \delta v_A + \min \{c, \delta (v'_B - v_A)\}.$$

It follows that

$$CS^P - CS = \delta (v'_B - v_A) - \min \{c, \delta (v'_B - v_A)\}.$$

Thus, bilateral data portability weakly increases consumer surplus in this case, and does so strictly whenever

$$c < \delta (v'_B - v_A).$$

The intuition is that without data portability, B would like to compensate the consumer for the future value it could create if it won in period 1. But if the required subsidy would involve a negative price, the non-negative price constraint prevents B from fully doing so. Bilateral data portability still improves B's period-2 competitive offering, so consumers can be better off.

The same possibility can arise in the intertemporal unbundling case

$$v'_B > v'_A > v_A > v_B.$$

If both firms offer data portability, A wins in period 1 and B wins in period 2. Neither firm has any reason to subsidize, so A sets its period 1 price equal to $v_A - v_B$, and B sets its period 2 price equal to $v'_B - v'_A$, leading to consumer surplus under bilateral data portability

$$CS^P = v_B + \delta v'_A.$$

Now suppose data portability is ruled out. A's constrained minimum price in period 1 is

$$p_A^m = \max \{0, c - \delta (v'_A - v_B)\}.$$

At this price, the maximum utility A can offer the consumer is

$$v_A + c - p_A^m + \delta v_B = v_A + \delta v_B + \min \{c, \delta (v'_A - v_B)\}$$

Similarly, the maximum utility B can offer the consumer is

$$v_B + \delta v_A + \min \{c, \delta (v'_B - v_A)\}.$$

The consumer obtains the lower of the two maximum utility offers, so

$$CS = \min \{v_A + \delta v_B + \min \{c, \delta (v'_A - v_B)\}, v_B + \delta v_A + \min \{c, \delta (v'_B - v_A)\}\}.$$

A simple sufficient condition for bilateral data portability to raise consumer surplus is then

$$c < \delta (v'_A - v_A).$$

To see this, note that the maximum utility B can offer without data portability is no greater than $v_B + \delta v_A + c$, and $c < \delta (v'_A - v_A)$ implies

$$v_B + \delta v_A + c < v_B + \delta v'_A = CS^P.$$

Since CS is no greater than B's maximum utility offer, it follows that $CS^P > CS$. Thus, once negative prices are ruled out, bilateral data portability can make forward-looking consumers better off relative to a regime in which data portability is unavailable.

C Non-personalized pricing example

Recall that we assume a consumer x who bought from A in period 1 is willing to pay $v'_A - \alpha_A x$ for firm A in period 2 and $\tilde{v}_B - \alpha_B(1 - x)$ for firm B, where $\tilde{v}_B = v'_B$ if A has chosen data portability, and $\tilde{v}_B = v_B$ if not. Similarly, a consumer who bought from B in period 1 is willing to pay $v'_B - \alpha_B(1 - x)$ for firm B in period 2 and $\tilde{v}_A - \alpha_A x$ for firm A, where $\tilde{v}_A = v'_A$ if B has chosen data portability, and $\tilde{v}_A = v_A$ if not. And we assume x is drawn from the uniform distribution over $[0, 1]$ independently of which firm the consumer bought from in period 1.

Start with those consumers who bought from A in period 1 in the case A does not offer data portability, and normalize their total measure to one. Then period-2 demand for firm A from these consumers is

$$\frac{v'_A - v_B + \alpha_B + p_B - p_A}{\alpha_A + \alpha_B},$$

so the period-2 profits for the two firms from these consumers are

$$\begin{aligned}\pi_A(A) &= \max_{p_A} \left\{ (p_A - c) \frac{v'_A - v_B + \alpha_B + p_B - p_A}{\alpha_A + \alpha_B} \right\} \\ \pi_B(A) &= \max_{p_B} \left\{ (p_B - c) \left(1 - \frac{v'_A - v_B + \alpha_B + p_B - p_A}{\alpha_A + \alpha_B} \right) \right\}.\end{aligned}$$

Straightforward calculations lead to

$$\pi_A(A) = \pi_A(v'_A, v_B), \quad \pi_B(A) = \pi_B(v'_A, v_B) \quad \text{and} \quad u(A) = u(v'_A, v_B),$$

where

$$\begin{aligned}\pi_A(\tilde{v}_A, \tilde{v}_B) &= \frac{(\alpha_A + 2\alpha_B + \tilde{v}_A - \tilde{v}_B)^2}{9(\alpha_A + \alpha_B)} \\ \pi_B(\tilde{v}_A, \tilde{v}_B) &= \frac{(2\alpha_A + \alpha_B - \tilde{v}_A + \tilde{v}_B)^2}{9(\alpha_A + \alpha_B)} \\ u(\tilde{v}_A, \tilde{v}_B) &= \frac{\tilde{v}_A + \tilde{v}_B}{2} - \frac{5}{8}(\alpha_A + \alpha_B) + \frac{(2(\tilde{v}_A - \tilde{v}_B) - (\alpha_A - \alpha_B))^2}{72(\alpha_A + \alpha_B)}.\end{aligned}$$

Similarly,

$$\begin{aligned}\pi_A(B) &= \pi_A(v_A, v'_B), \quad \pi_B(B) = \pi_B(v_A, v'_B) \quad \text{and} \quad u(B) = u(v_A, v'_B) \\ \pi_A^P &= \pi_A(v'_A, v'_B), \quad \pi_B^P = \pi_B(v'_A, v'_B) \quad \text{and} \quad u^P = u(v'_A, v'_B).\end{aligned}$$

To ensure all of the relevant solutions are interior so that these expressions are valid, we need to

assume

$$-\alpha_A - 2\alpha_B < v_A - v'_B < v'_A - v_B < 2\alpha_A + \alpha_B,$$

which also implies

$$-\alpha_A - 2\alpha_B < v'_A - v'_B < 2\alpha_A + \alpha_B.$$

With forward-looking consumers, we are interested in total surplus. We have

$$\begin{aligned} S(\tilde{v}_A, \tilde{v}_B) &\equiv \pi_A(\tilde{v}_A, \tilde{v}_B) + \pi_B(\tilde{v}_A, \tilde{v}_B) + u(\tilde{v}_A, \tilde{v}_B) \\ &= \frac{\tilde{v}_A + \tilde{v}_B}{2} - \frac{1}{8}(\alpha_A + \alpha_B) + \frac{5(2(\tilde{v}_A - \tilde{v}_B) - (\alpha_A - \alpha_B))^2}{72(\alpha_A + \alpha_B)}. \end{aligned}$$

So A wants to offer data portability iff $S(v'_A, v'_B) > S(v'_A, v_B)$.

We have

$$\frac{\partial S(v'_A, v_B)}{\partial v_B} = \frac{1}{2} - \frac{5(2(v'_A - v_B) - (\alpha_A - \alpha_B))}{18(\alpha_A + \alpha_B)},$$

so $\frac{\partial S(v'_A, v_B)}{\partial v_B}$ is increasing in v_B and $\frac{\partial S(v'_A, v_B)}{\partial v_B} \geq 0$ is equivalent to

$$v'_A - v_B \leq \frac{7\alpha_A + 2\alpha_B}{5}.$$

Thus, if $v'_A - v_B \leq \frac{7\alpha_A + 2\alpha_B}{5}$, then $S(v'_A, v)$ is increasing in v for $v_B \leq v \leq v'_B$, so $S(v'_A, v'_B) > S(v'_A, v_B)$.

If on the other hand

$$\frac{7\alpha_A + 2\alpha_B}{5} < v'_A - v'_B < v'_A - v_B < 2\alpha_A + \alpha_B,$$

then $S(v'_A, v)$ is decreasing in v for $v_B \leq v \leq v'_B$, so $S(v'_A, v'_B) < S(v'_A, v_B)$.

With myopic consumers, we are interested in joint profits, and it is straightforward to verify that

$$\pi_A(v'_A, v'_B) + \pi_B(v'_A, v'_B) > \pi_A(v'_A, v_B) + \pi_B(v'_A, v_B)$$

is equivalent to

$$v'_A < \frac{v_B + v'_B + \alpha_A - \alpha_B}{2}.$$

And

$$\pi_A(v'_A, v'_B) + \pi_B(v'_A, v'_B) > \pi_A(v_A, v'_B) + \pi_B(v_A, v'_B)$$

is equivalent to

$$v'_B < \frac{v_A + v'_A + \alpha_B - \alpha_A}{2}.$$

D General proof of Proposition 1

Here we provide a more general proof of Proposition 1, covering the possibility that v_A and/or v_B are negative.

Specifically, we prove that starting from a situation in which neither firm has committed to DP, each firm prefers to unilaterally deviate to DP, and that starting from a situation in which one firm has committed to DP but the other has not, the latter has an incentive to deviate to DP.

Lemma 2. *If $v_A \geq v'_B$, then $\Pi_A = V_A - \max\{V_B, 0\}$ and $\Pi_B = 0$, regardless of the DP regime. And if $v_B \geq v'_A$, then $\Pi_A = 0$ and $\Pi_B = V_B - \max\{V_A, 0\}$, regardless of the DP regime.*

Proof of Lemma 2 Suppose $v_A \geq v'_B$. This means A wins period 2 no matter the outcome in period 1, so we have $u(A) = \max\{v_B, 0\}$ and $u(B) = u^P = v'_B$.

Since B never wins in period 2, it is unwilling to subsidize in period 1, so the highest PDV of utility that B can offer the consumer in period 1 is $v_B + \delta v'_B$, by setting $p_B = c$. The outside option for the consumer in period 1 is to join neither firm, in which case she will end up joining firm A in period 2, obtaining a PDV of utility $\delta \max\{v_B, 0\}$. Note that $v'_B > 0$ implies that

$$\max\{v_B + \delta v'_B, \delta \max\{v_B, 0\}\} = \max\{v_B + \delta v'_B, 0\} = \max\{V_B, 0\}.$$

Thus, to win the consumer in period 1, A sets p_A such that

$$v_A - (p_A - c) + \delta u(A) = \max\{V_B, 0\},$$

leading to profits

$$\begin{aligned} \Pi_A &= (p_A - c) + \delta (v'_A - u(A)) \\ &= v_A + \delta u(A) - \max\{V_B, 0\} + \delta (v'_A - u(A)) \\ &= V_A - \max\{V_B, 0\} > 0. \end{aligned}$$

Since this is positive, A wins both periods and $\Pi_B = 0$.

The result for the case $v_B \geq v'_A$ follows by symmetry in A and B.

■

Thus, throughout the rest of this proof, we focus on the case $v'_A > v_B$ and $v'_B > v_A$. This means that if a firm does not offer DP and wins in period 1, it will also win in period 2.

We first determine the equilibrium for the case when neither firm has committed to DP.

Lemma 3. *Without DP, the equilibrium outcome is as follows. If $V_A \geq V_B$, then firm A attracts the consumer in both periods, resulting in profits and consumer surplus*

$$\begin{aligned}\Pi_A &= V_A - \max\{V_B, 0\} \\ \Pi_B &= 0.\end{aligned}$$

The results for the case when $V_B \geq V_A$ are obtained by symmetry in A and B.

Proof of Lemma 3. If $V_B \leq 0$, then B cannot exert any competitive pressure on A in either period, so the binding constraint is the outside option, meaning that $\Pi_A = V_A$. To see this, note that if B does not win the consumer in the first period, then B is irrelevant in the second period because $v_B < 0$. This implies that the lowest price p_B that B is willing to offer in the first period is such that $p_B - c = -\delta(v'_B - \max\{v_A, 0\})$, which means the highest PDV of net utility that B can offer in the first period is

$$v_B + \delta(v'_B - \max\{v_A, 0\}) + \delta \max\{v_A, 0\} = V_B \leq 0.$$

Thus, B is also irrelevant in the first period.

Suppose then $V_A > V_B > 0$. First, we show that in equilibrium the consumer must join one of the two firms in period 1. Suppose she doesn't. Then her PDV of utility is $\delta \max\{\min\{v_A, v_B\}, 0\}$ and firm A makes zero profits. But firm A could attract the consumer in period 1 (and therefore also in period 2) by setting p_A such that

$$v_A - (p_A - c) + \delta \max\{v_B, 0\} = \delta \max\{\min\{v_A, v_B\}, 0\},$$

leading to total profits

$$p_A - c + \delta(v'_A - \max\{v_B, 0\}) = V_A - \delta \max\{\min\{v_A, v_B\}, 0\} > 0.$$

Second, the consumer cannot join firm B in equilibrium. If she did, then firm B attracts the consumer in both periods, so the utility derived by the consumer from the perspective of period 1 is $v_B - (p_B - c) + \delta \max\{v_A, 0\}$. Firm A's profits are zero and firm B's profits are $p_B - c + \delta(v'_B - \max\{v_A, 0\})$, which must be non-negative. This implies

$$v_B - (p_B - c) + \delta \max\{v_A, 0\} \leq V_B.$$

But then firm A could attract the consumer in period 1 (and therefore also in period 2) by setting

p_A such that

$$v_A - (p_A - c) + \delta \max \{v_B, 0\} = V_B,$$

leading to total profits

$$p_A - c + \delta (v'_A - \max \{v_B, 0\}) = V_A - V_B \geq 0.$$

Thus, A must win the consumer in both periods and its price p_A in the first period must be such that

$$v_A - (p_A - c) + \delta \max \{v_B, 0\} = \max \{V_B, \delta \max \{\min \{v_A, v_B\}, 0\}\} = V_B.$$

This means A's profit is $\Pi_A = V_A - V_B$, while consumer surplus is V_B .

■

Now consider the case when both firms have committed to DP (or DP has been imposed by policy).

Lemma 4. *Suppose both firms have committed to DP. If $v'_A \geq v'_B$ and ($V_B < 0$ or $v_A \geq v_B$), then A wins both periods, resulting in profits and consumer surplus*

$$\begin{aligned} \Pi_A^P &= V_A - \max \{V_B, 0\} \\ \Pi_B^P &= 0. \end{aligned}$$

If $v'_A \geq v'_B$ and $V_B \geq 0$ and $v_B > v_A$, then B wins the first period and A wins the second period, resulting in profits and consumer surplus

$$\begin{aligned} \Pi_A^P &= \delta (v'_A - v'_B) \\ \Pi_B^P &= \min \{v_B - v_A, V_B\}. \end{aligned}$$

The results for the case $v'_A \leq v'_B$ are obtained by symmetry in A and B.

Proof of Lemma 4 Suppose both firms have committed to DP, and recall $v'_A > v_B$, $v'_B > v_A$. In equilibrium, the consumer must buy from one of the two firms in period 1. If not, then she would obtain PDV of utility $\delta \max \{\min \{v_A, v_B\}, 0\}$ and firm $i \in \{A, B\}$ such that $V_i > 0$ (recall this must be true for $i = A$ or $i = B$) could profitably deviate by setting its first-period price p_i such that

$$v_i - (p_i - c) + \delta \min \{v'_A, v'_B\} = \delta \max \{\min \{v_A, v_B\}, 0\},$$

yielding profits

$$p_i - c + \delta \max \{v'_i - v'_j, 0\} = V_i - \delta \max \{\min \{v_A, v_B\}, 0\} > 0.$$

Since the outcome in period 2 (firm A wins if $v'_A \geq v'_B$ and firm B wins otherwise) does not depend on who wins in period 1, neither firm is willing to subsidize in period 1.

Suppose $v'_A \geq v'_B$, so A wins in period 2 regardless of who wins in period 1. If A also wins in period 1, then we must have

$$v_A - (p_A - c) + \delta v'_B = \max \{V_B, \max \{\min \{v_A, v_B\}, 0\}\} = \max \{V_B, 0\}.$$

This implies

$$\begin{aligned} \Pi_A^P &= V_A - \max \{V_B, 0\} \\ \Pi_B^P &= 0 \end{aligned}$$

and this solution is valid only if $V_A \geq \max \{V_B, 0\}$ and A does not want to deviate by slightly lowering p_A . This deviation can only be profitable if $V_B \geq 0$, in which case the deviation yields profits of $\delta(v'_A - v'_B)$. Thus, the deviation is not profitable if $V_B < 0$ or $v_A - v_B \geq 0$. So this solution is valid if and only if $V_B < 0$ or $v_A - v_B \geq 0$ (note indeed, that either one of these inequalities implies $V_A \geq \max \{V_B, 0\}$ because we are in the case $v'_A \geq v'_B$).

If B wins period 1, then we must have

$$v_B - (p_B - c) + \delta v'_B = \max \{v_A + \delta v'_B, 0\},$$

so B's profits are $V_B - \max \{v_A + \delta v'_B, 0\}$. This solution is therefore valid if and only if (iff) $V_B \geq 0$ and $v_B - v_A \geq 0$, in which case

$$\begin{aligned} \Pi_A^P &= \delta(v'_A - v'_B) \\ \Pi_B^P &= \min \{v_B - v_A, V_B\}. \end{aligned}$$

■

Finally, suppose only one firm (A) has committed to DP.

Lemma 5. *Suppose firm A has committed to DP, while firm B has not. If $v'_A \geq v'_B$, then the*

outcome is identical to the one without DP, so

$$\begin{aligned}\Pi_A^{PA} &= \max \{V_A - \max \{V_B, 0\}, 0\} \\ \Pi_B^{PA} &= \max \{V_B - \max \{V_A, 0\}, 0\}.\end{aligned}$$

If $v'_B > v'_A$, then the outcome is the same as with bilateral DP, so

$$\begin{aligned}\Pi_A^{PA} &= 0 \\ \Pi_B^{PA} &= V_B - \max \{V_A, 0\}\end{aligned}$$

when $V_A \leq 0$ or $v_B - v_A \geq 0$, and

$$\begin{aligned}\Pi_A^{PA} &= \min \{v_A - v_B, V_A\} \\ \Pi_B^{PA} &= \delta (v'_B - v'_A)\end{aligned}$$

when $V_A \geq 0$ and $v_A - v_B \geq 0$.

Proof of Lemma 5 Suppose firm A has committed to DP, while firm B has not. In equilibrium, the consumer buys from one of the two firms in period 1. If not, then she would obtain PDV of utility $\delta \max \{\min \{v_A, v_B\}, 0\}$ and firm A could profitably deviate by setting its first-period price p_A such that

$$v_A - (p_A - c) + \delta \min \{v'_A, v'_B\} = \delta \max \{\min \{v_A, v_B\}, 0\},$$

yielding profits

$$p_A - c + \delta \max \{v'_A - v'_B, 0\} = V_A - \delta \max \{\min \{v_A, v_B\}, 0\} > 0.$$

If $v'_A \geq v'_B$, then whichever firm wins period 1 also wins period 2. In this case, the analysis is the same as with no DP, leading to

$$\begin{aligned}\Pi_A^{PA} &= \max \{V_A - \max \{V_B, 0\}, 0\} \\ \Pi_B^{PA} &= \max \{V_B - \max \{V_A, 0\}, 0\}.\end{aligned}$$

If $v'_B > v'_A$, then firm B wins period 2, regardless of who wins period 1. In this case, the analysis is the same as with bilateral DP, so profits and consumer surplus are the same as in Lemma 4 for the case $v'_B > v'_A$.

■

Start with no DP by either firm and suppose $V_A \geq \max \{V_B, 0\}$, so A wins in the absence of

any DP and profits are

$$\begin{aligned}\Pi_A &= V_A - \max\{V_B, 0\} \\ \Pi_B &= 0.\end{aligned}$$

Then, from Lemma 5, if $v'_A \geq v'_B$, the outcome with unilateral DP by A is the same as without DP.

If $v'_A < v'_B$ then we must have $v_A > v_B$, so profits are

$$\begin{aligned}\Pi_A^{PA} &= \min\{v_A - v_B, V_A\} \\ \Pi_B^{PA} &= \delta(v'_B - v'_A)\end{aligned}$$

It is easily seen that B's profits are strictly higher than without DP. As for A, if $V_B < 0$, this implies

$$v_B + \delta v'_A < v_B + \delta v'_B < 0,$$

so A's profits are V_A in both cases. And if $V_B \geq 0$, then A's profits are strictly higher with unilateral DP by A because

$$\min\{v_A - v_B, V_A\} > V_A - V_B.$$

Thus, A has an incentive to offer unilateral DP, and strictly so provided $v'_A < v'_B$ and $V_B \geq 0$.

Now compare the profits under unilateral DP by A (lemma 5) to the profits under bilateral DP (lemma 4). If $v'_B > v'_A$, then the outcome and firm profits are the same under unilateral DP by A and bilateral DP.

If $v'_A \geq v'_B$, then profits under unilateral DP by A are

$$\begin{aligned}\Pi_A^{PA} &= V_A - \max\{V_B, 0\} \\ \Pi_B^{PA} &= 0.\end{aligned}$$

If $v'_A \geq v'_B$ and ($V_B < 0$ or $v_A \geq v_B$), these are also the profits under bilateral DP.

If $v'_A \geq v'_B$ and $V_B \geq 0$ and $v_B \geq v_A$, then profits under bilateral DP are

$$\begin{aligned}\Pi_A^P &= \delta(v'_A - v'_B) \\ \Pi_B^P &= \min\{v_B - v_A, V_B\}.\end{aligned}$$

Clearly, B's profits are higher, strictly so if $v'_A \geq v'_B$ and $V_B > 0$ and $v_B > v_A$. And A's profits are also higher in this case, because $\delta(v'_A - v'_B) \geq V_A - V_B$.

In summary, firm A does at least as well and in some cases strictly better by deviating to

unilateral DP from the situation without any DP. And firm B does at least as well and in some cases strictly better by deviating from the case with unilateral DP by A to the case with bilateral DP. Thus, provided there is a positive measure of consumers with $v_A > v_B$ and $v'_A < v'_B$ and a positive measure with $v_A < v_B$ and $v'_A > v'_B$, the unique equilibrium has both firms choosing to commit to DP.

The result for the case $V_B \geq \max\{V_A, 0\}$ follows by symmetry in A and B.