

Online Appendix

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This Online Appendix supplements the proofs in the main appendix of the paper “Personalization, data portability and competitive dynamics”. Section A covers the baseline results in Section 4 of the paper while Section B covers the case with myopic consumers in Section 5 of the paper. Sections C and D analyze the personalized pricing and imperfect data portability extensions in Sections 6.1 and 6.2, respectively.

Throughout this appendix, we replace “data portability” with “DP” and define $\alpha = \alpha_A + \alpha_B$ and $\beta = \beta_A + \beta_B$. These measure the rates at which a consumer’s relative utility changes with its location in periods 1 and 2, respectively.

A Proofs of results in Section 4

A.1 Full proof of Lemma 1

Fix the firms’ DP choices. The assumptions on continuation payoffs imply that $\pi_i^d(j)$ and $u^d(j)$ are constants when firms choose their initial prices. The indifferent consumer’s location is

$$x = \frac{v_A - v_B + \alpha_B + p_B - p_A + \delta[u^d(A) - u^d(B)]}{\alpha}.$$

Firm A’s discounted profit is

$$\Pi_A = [p_A - c_A + \delta\{\pi_A^d(A) - \pi_A^d(B)\}] F(x) + \delta\pi_A^d(B),$$

and B’s analogous expression uses $1 - F(x)$ and $\pi_B^d(B) - \pi_B^d(A)$. The term in square brackets is A’s initial price-cost margin plus the discounted change in its continuation profit from acquiring a customer rather than leaving that customer with B.

For $0 < x < 1$, define

$$h_A(x) = \frac{F(x)}{f(x)}, \quad h_B(x) = \frac{1 - F(x)}{f(x)}, \quad L(x) = \alpha\{x + h_A(x) - h_B(x)\} - \alpha_B.$$

Log-concavity of F and $1 - F$ gives $h'_A(x) \geq 0$ and $h'_B(x) \leq 0$. In particular, L is strictly increasing.

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Differentiating each firm's profit with respect to its price yields

$$\begin{aligned} p_A - c_A + \delta\{\pi_A^d(A) - \pi_A^d(B)\} &= \alpha h_A(x), \\ p_B - c_B + \delta\{\pi_B^d(B) - \pi_B^d(A)\} &= \alpha h_B(x). \end{aligned} \tag{10}$$

Substituting these first-order conditions into the expression for the indifferent location gives

$$L(x) = \Omega^d, \quad \Omega^d = w_A - w_B + \delta\{S^d(A) - S^d(B)\},$$

where $w_i = v_i - c_i$. The maintained interiority assumption means that Ω^d lies strictly between the endpoint limits of L , so the conditions determine a unique interior solution, denoted $x(\Omega^d)$.

The first-order conditions give global best responses. For fixed p_B , on the set where A's square-bracketed term and demand are both positive, the logarithm of their product is strictly concave in p_A . This follows because the logarithm of the positive affine margin term is strictly concave and $\log F(x)$ is concave in p_A . The analogous argument for B uses log-concavity of $1 - F$. Thus the first-order conditions maximize each firm's profit even against deviations that capture the entire market. A nonpositive square-bracketed term gives nonpositive incremental profit and cannot improve on the positive incremental profit at the interior solution.

Define

$$H_A(\Omega) = \alpha \frac{F(x(\Omega))^2}{f(x(\Omega))}, \quad H_B(\Omega) = \alpha \frac{[1 - F(x(\Omega))]^2}{f(x(\Omega))}.$$

The first-order conditions now give

$$\Pi_A = H_A(\Omega^d) + \delta\pi_A^d(B), \quad \Pi_B = H_B(\Omega^d) + \delta\pi_B^d(A).$$

Since F is strictly increasing, h_A is nondecreasing and positive, and $x(\Omega)$ is strictly increasing, H_A is strictly increasing. Similarly, H_B is strictly decreasing. More explicitly,

$$\begin{aligned} H'_A(\Omega) &= \frac{F(x)[1 + h'_A(x)]}{1 + h'_A(x) - h'_B(x)} \in (0, 1), \\ -H'_B(\Omega) &= \frac{[1 - F(x)][1 - h'_B(x)]}{1 + h'_A(x) - h'_B(x)} \in (0, 1). \end{aligned} \tag{11}$$

A.2 Proof of firms' discounted profits in uniform case

If F is uniform, $h_A(x) = x$ and $h_B(x) = 1 - x$, so

$$x = s_A = \frac{\alpha_A + 2\alpha_B + \Omega^d}{3\alpha}.$$

Substitution into (10) and multiplication by the corresponding shares give $H_A = \alpha s_A^2$ and $H_B = \alpha(1 - s_A)^2$, which yield the stated uniform-demand profit formulas in the text after Lemma 1.

A.3 Proof of Corollary 1

For this proof, define $\tilde{w}_i = \tilde{v}_i - c_i$. Consider a customer group with net values $(\tilde{w}_A, \tilde{w}_B)$ before location terms. Write $\tilde{s}_i$ for firm i 's period-2 share of this group. With a uniform period-2 location draw and both firms serving the group, the pricing argument above, applied to period 2, gives A's equilibrium share as

$$\tilde{s}_A = \frac{\beta_A + 2\beta_B + \tilde{w}_A - \tilde{w}_B}{3\beta}.$$

Expected total surplus per consumer in the group is

$$\begin{aligned} S(\tilde{w}_A, \tilde{w}_B) &= \int_0^{\tilde{s}_A} (\tilde{w}_A - \beta_A y) dy + \int_{\tilde{s}_A}^1 [\tilde{w}_B - \beta_B(1 - y)] dy \\ &= \frac{\tilde{w}_A + \tilde{w}_B}{2} - \frac{\beta}{8} + \frac{5[2(\tilde{w}_A - \tilde{w}_B) - (\beta_A - \beta_B)]^2}{72\beta}. \end{aligned} \tag{12}$$

The maintained assumption of positive period-2 shares for both firms, with and without DP, is equivalent to

$$-\beta_A - 2\beta_B < w_A - w'_B < w'_A - w_B < 2\beta_A + \beta_B.$$

The data-enabled difference $w'_A - w'_B$ lies strictly between the two no-DP differences. Thus, positive shares for both firms in the two no-DP customer groups also ensure positive shares with DP.

A's DP changes the net values for its original customers from (w'_A, w_B) to (w'_A, w'_B) ; B's DP changes them from (w_A, w'_B) to (w'_A, w'_B) . Subtracting the corresponding surplus expressions gives

$$\begin{aligned} S^P - S(A) &= \frac{5(v'_B - v_B)}{9\beta} \left[\frac{7\beta_A + 2\beta_B}{5} - w'_A + \frac{w_B + w'_B}{2} \right], \\ S^P - S(B) &= \frac{5(v'_A - v_A)}{9\beta} \left[\frac{2\beta_A + 7\beta_B}{5} - w'_B + \frac{w_A + w'_A}{2} \right]. \end{aligned}$$

Since $v'_i - v_i > 0$ and $\beta > 0$, Proposition 1 gives the two exact conditions in the text of Corollary 1, including the equality and reversed-inequality cases.

Now suppose $\beta_A = \beta_B = t > 0$, $w_A > w_B$, and $w'_B > w'_A$. Positive second-period shares without DP imply $w'_A - w_B < 3t$ and $w'_B - w_A < 3t$. Therefore,

$$w'_A - \frac{w_B + w'_B}{2} = \frac{(w'_A - w_B) + (w'_A - w'_B)}{2} < \frac{3t}{2} < \frac{9t}{5}.$$

Thus A strictly prefers to offer DP. If also $0 < w'_B - w'_A \leq 3t/5$, then

$$w'_B - \frac{w_A + w'_A}{2} = \frac{(w'_B - w_A) + (w'_B - w'_A)}{2} < \frac{3t + 3t/5}{2} = \frac{9t}{5}.$$

B therefore strictly prefers to offer DP as well. Each firm's strict preference is independent of the rival's decision, so bilateral DP is the unique equilibrium in DP decisions.

More generally, at least one firm strictly prefers to offer DP. Otherwise, weakly reversing both offering inequalities and adding them would give

$$(v'_A - v_A) + (v'_B - v_B) \geq \frac{18}{5}\beta.$$

But positive second-period shares in the two customer groups without DP require

$$w'_A - w_B < 2\beta_A + \beta_B, \quad w'_B - w_A < \beta_A + 2\beta_B.$$

Adding these gives

$$(v'_A - v_A) + (v'_B - v_B) < 3\beta,$$

a contradiction. Thus NN cannot be an equilibrium.

A.4 Proof of Corollary 2

Suppose $w'_B - w'_A \geq \beta_A + 2\beta_B$. With DP, the period-2 equilibrium has A pricing at cost and B charging $c_B + w'_B - w'_A - \beta_B$, serving the entire customer group. Indeed, starting from these prices, A cannot attract consumers at a price at or above cost. If B lowers its price, its market share stays unchanged, and if B raises it by $\varepsilon < \beta$, the resulting profit change is

$$(w'_B - w'_A - \beta_B + \varepsilon) \left(1 - \frac{\varepsilon}{\beta}\right) - (w'_B - w'_A - \beta_B) = \varepsilon \left(1 - \frac{w'_B - w'_A - \beta_B}{\beta}\right) - \frac{\varepsilon^2}{\beta} < 0$$

because $w'_B - w'_A - \beta_B \geq \beta$.

Moreover, at every $y \in [0, 1]$,

$$w'_B - \beta_B(1 - y) - (w'_A - \beta_A y) = w'_B - w'_A - \beta_B + \beta y \geq \beta > 0.$$

Thus, among A's original customers, those who move from A to B strictly gain in net value, while those already buying from B gain $v'_B - v_B > 0$. Consequently, $S^P > S(A)$. B also serves all its own original customers without DP, since $w_A < w'_A$, so $S^P = S(B)$. Proposition 1 implies that A strictly prefers to offer DP and B is indifferent, regardless of the rival's choice.

The symmetric argument applies when $w'_A - w'_B \geq 2\beta_A + \beta_B$. In each case, both firms offering DP and only the strictly willing firm offering DP are equilibria, whereas NN is not. For $\beta_A = \beta_B = t$, the two whole-market conditions reduce to $|w'_A - w'_B| \geq 3t$.

A.5 Consumer-surplus decomposition

First suppose only that F is uniform. A consumer at initial location z obtains lifetime net utility $v_B - \alpha_B(1 - z) - p_B + \delta u^d(B)$ from buying from B initially. Consumers with $z < s_A^d$ instead buy from A, gaining an additional $\alpha(s_A^d - z)$ relative to buying from B. Integrating and substituting B's equilibrium price from (10) gives

$$\begin{aligned} CS^d = w_B - \frac{\alpha_B}{2} + \delta S^d(B) - \delta[\pi_A^d(B) + \pi_B^d(A)] \\ + \alpha \left[\frac{(s_A^d)^2}{2} + s_A^d - 1 \right]. \end{aligned} \quad (13)$$

Moreover, $s_A^d = \frac{\alpha_A + 2\alpha_B + \Omega^d}{3\alpha}$ implies

$$s_A^{PP} - s_A^{NN} = \frac{\delta[S(B) - S(A)]}{3\alpha}.$$

Subtracting (13) under NN from its value under PP and using this expression for the difference in market shares gives (5). Thus the decomposition includes the full change in first-period prices and market shares; it holds for any second-period payoffs satisfying the model's independence assumptions.

A.6 Proof of Proposition 2

In addition to uniform F , impose uniform G , unchanged differentiation across periods, and positive shares in each period-2 customer group. Thus $\beta_i = \alpha_i$ for each firm. Define

$$\begin{aligned} \Sigma &= \frac{(v'_A - v_A) + (v'_B - v_B)}{\alpha}, & \Delta &= \frac{(v'_A - v_A) - (v'_B - v_B)}{\alpha}, \\ k &= \frac{w_A + w'_A - w_B - w'_B - (\alpha_A - \alpha_B)}{\alpha}. \end{aligned}$$

Here Σ measures the combined utility improvement from data relative to product differentiation, while Δ measures the difference in the firms' improvements on the same scale; $\Delta > 0$ means A improves more. The expression k measures A's average advantage across the cases without and with data, allowing for unequal differentiation. Specifically, $k/2$ is A's average net-value advantage at location $1/2$ across these two cases, divided by α . These expressions will be used repeatedly

throughout the rest of this proof as well as the proof of Proposition 4. Positive data gains imply $\Sigma > 0$ and $|\Delta| < \Sigma$.

In a period-2 group with net values $(\tilde{w}_A, \tilde{w}_B)$, A's period-2 share and the two per-consumer profits are

$$\tilde{s}_A = \frac{1}{2} + \frac{2(\tilde{w}_A - \tilde{w}_B) - (\alpha_A - \alpha_B)}{6\alpha}, \quad \pi_A = \alpha \tilde{s}_A^2, \quad \pi_B = \alpha(1 - \tilde{s}_A)^2.$$

Apply these formulas and the surplus formula (12) in the proof of Corollary 1 to the three pairs (w'_A, w_B) , (w_A, w'_B) , and (w'_A, w'_B) . They give

$$\begin{aligned} S(A) - S(B) &= \alpha \left[\frac{\Delta}{2} + \frac{5\Sigma k}{18} \right], \\ S^P - \frac{S(A) + S(B)}{2} &= \alpha \left[\frac{\Sigma}{4} + \frac{5}{36} \left(k\Delta + \frac{\Delta^2 - \Sigma^2}{2} \right) \right], \\ [\pi_B^P - \pi_B(A)] + [\pi_A^P - \pi_A(B)] &= \alpha \left[\frac{\Sigma}{3} + \frac{1}{9} \left(k\Delta + \frac{\Delta^2 - \Sigma^2}{2} \right) \right]. \end{aligned}$$

Substituting into (5) and simplifying yields

$$\begin{aligned} \frac{CS^{PP} - CS^{NN}}{\delta\alpha} &= -\frac{\Sigma}{12} + \frac{3\Delta^2 - \Sigma^2}{72} - \frac{5\Sigma k(k - \Delta)}{324} \\ &\quad - \frac{\delta}{18} \left(\frac{\Delta}{2} + \frac{5\Sigma k}{18} \right)^2. \end{aligned} \tag{14}$$

For condition (i), use $k(k - \Delta) = (k - \Delta/2)^2 - \Delta^2/4$ and $\Delta^2 \leq \Sigma^2$ to obtain

$$\frac{CS^{PP} - CS^{NN}}{\delta\alpha} \leq \Sigma \left[-\frac{1}{12} + \frac{\Sigma}{36} + \frac{5\Sigma^2}{1296} \right].$$

The bracket is increasing in Σ and equals $-1/81$ at $\Sigma = 2$. Thus $\Sigma \leq 2$ implies $CS^{PP} - CS^{NN} < 0$. By the definition of Σ , this is condition (i) in the text.

For condition (ii), A's period-2 shares in the two no-DP customer groups are $1/2 + (k + \Sigma)/6$ and $1/2 + (k - \Sigma)/6$. Positive shares for both firms therefore require $|k| + \Sigma < 3$, so $0 < \Sigma < 3$. The two weak offering conditions from Corollary 1 are equivalent to

$$k + \frac{\Delta + \Sigma}{2} \leq \frac{9}{5}, \quad k + \frac{\Delta - \Sigma}{2} \geq -\frac{9}{5},$$

or $|k + \Delta/2| \leq 9/5 - \Sigma/2$. Dropping the final, nonpositive term in (14) gives

$$\frac{CS^{PP} - CS^{NN}}{\delta\alpha} \leq -\frac{\Sigma}{12} - \frac{\Sigma^2}{72} + \frac{\Delta^2}{24} + \frac{5\Sigma k\Delta}{324} - \frac{5\Sigma k^2}{324}.$$

For an upper bound, retain only $0 < \Sigma < 3$, $|\Delta| \leq \Sigma$, and the displayed offering restriction. Holding $k + \Delta/2$ fixed, the right-hand side is quadratic in Δ with coefficient $(18 - 5\Sigma)/432 > 0$. Its maximum over $|\Delta| \leq \Sigma$ is therefore at $\Delta = \Sigma$ or $\Delta = -\Sigma$. The bound and the restrictions are unchanged when the signs of both k and Δ are reversed, so it suffices to take $\Delta = \Sigma$. The offering restriction then becomes $-9/5 \leq k \leq 9/5 - \Sigma$. The bound is concave in k and is maximized at $k = \min\{\Sigma/2, 9/5 - \Sigma\}$. Consequently,

$$\frac{CS^{PP} - CS^{NN}}{\delta\alpha} \leq \begin{cases} \Sigma \left[-\frac{1}{12} + \frac{\Sigma}{36} + \frac{5\Sigma^2}{1296} \right], & 0 < \Sigma \leq 6/5, \\ -\Sigma \left[\frac{1}{30} + \frac{5}{162} \left(\Sigma - \frac{9}{5} \right)^2 \right], & 6/5 \leq \Sigma < 3. \end{cases}$$

In the first case the bracket is at most $-2/45$; the second case is bounded above by $-\Sigma/30$. We obtain the common bound

$$CS^{PP} - CS^{NN} \leq -\frac{\delta\alpha\Sigma}{30} = -\frac{\delta}{30} [(v'_A - v_A) + (v'_B - v_B)] < 0.$$

This proves condition (ii), including the case with weak offering incentives. A unique bilateral-DP equilibrium requires both incentives to be strict and is therefore covered as well.

A.7 Effect of a portability mandate

Under the assumptions of Proposition 2, we establish the subsequent claim in the text that a mandate cannot benefit consumers when firms can credibly commit to portability. For this comparison, (13) also applies under unilateral DP. Suppose NP is a voluntary equilibrium, so B offers DP and A withholds it. Proposition 1 implies $S(A) \geq S^P$, and therefore

$$s_A^{PP} - s_A^{NP} = \frac{\delta[S^P - S(A)]}{3\alpha} \leq 0.$$

Requiring A to offer DP leaves $S^d(B) = S^P$ and $\pi_A^d(B) = \pi_A^P$ unchanged. It strictly increases B's second-period share of A's original customers and hence its profit: $\pi_B^P > \pi_B(A)$. Subtracting (13) under NP from its value under PP gives

$$CS^{PP} - CS^{NP} = -\delta[\pi_B^P - \pi_B(A)] + \alpha(s_A^{PP} - s_A^{NP}) \left[1 + \frac{s_A^{PP} + s_A^{NP}}{2} \right] < 0.$$

The argument for a voluntary PN equilibrium is symmetric. Corollary 1 rules out a voluntary NN equilibrium under these assumptions. Thus, when firms can credibly commit, a mandate is either redundant or strictly reduces consumer surplus.

B Proofs of results in Section 5

B.1 Proof of Corollary 3

For period-2 net values $(\tilde{w}_A, \tilde{w}_B)$, the uniform pricing formulas give joint profit per consumer

$$\frac{\beta}{2} + \frac{[2(\tilde{w}_A - \tilde{w}_B) - (\beta_A - \beta_B)]^2}{18\beta}. \quad (15)$$

Each firm's equilibrium price-cost margin equals β times its share of the group. Joint profit is therefore lowest when the shares are equal and rises with the gap between them. Subtracting the no-DP profits for each initial customer group from the profits with DP gives

$$\begin{aligned} \pi_A^P + \pi_B^P - \pi_A(A) - \pi_B(A) &= \frac{2(v'_B - v_B)}{9\beta} [\beta_A - \beta_B - 2w'_A + w_B + w'_B], \\ \pi_A^P + \pi_B^P - \pi_A(B) - \pi_B(B) &= \frac{2(v'_A - v_A)}{9\beta} [\beta_B - \beta_A - 2w'_B + w_A + w'_A]. \end{aligned}$$

The strictly positive data gains establish the strict offering conditions, with equality giving indifference. Even their weak versions cannot both hold: adding them would require $w'_A + w'_B \leq w_A + w_B$. Independence from the rival's decision gives the equilibrium classification, including indifference when an offering condition holds with equality. These calculations use uniformity of G only. Proposition 3 holds for any F satisfying the baseline assumptions, so no additional restriction on F is needed.

B.2 Comparison with forward-looking consumers

Allowing indifference, A's myopic condition can be written $w'_A - (w_B + w'_B)/2 \leq (\beta_A - \beta_B)/2$. The corresponding forward-looking bound in Corollary 1 is greater by $9\beta/10 > 0$. The same difference applies to B's bound. Thus any firm weakly willing to offer DP with myopic consumers strictly prefers to offer it with forward-looking consumers.

B.3 Consumer-surplus decomposition and price changes

Use Σ , Δ , and k as defined in the proof of Proposition 2. Thus $\Sigma > 0$ and $|\Delta| < \Sigma$. We allow unequal differentiation parameters across firms throughout this derivation, while retaining unchanged differentiation across periods.

With uniform F , the initial price formulas are

$$\begin{aligned} p_A^d - c_A &= \alpha s_A^d - \delta[\pi_A^d(A) - \pi_A^d(B)], \\ p_B^d - c_B &= \alpha(1 - s_A^d) - \delta[\pi_B^d(B) - \pi_B^d(A)]. \end{aligned}$$

Integrating period-1 net utility and adding expected discounted net utility in period 2 gives

$$\begin{aligned} CS^d &= w_B - \frac{\alpha_B}{2} + \alpha \left[\frac{(s_A^d)^2}{2} + s_A^d - 1 \right] + \delta[\pi_B^d(B) - \pi_B^d(A)] \\ &\quad + \delta[s_A^d u^d(A) + (1 - s_A^d) u^d(B)]. \end{aligned}$$

The uniform period-2 formulas imply

$$\begin{aligned} [\pi_A(A) - \pi_A(B)] + [\pi_B(B) - \pi_B(A)] &= \frac{2\alpha\Sigma}{3}, \\ [\pi_A(A) + \pi_B(A)] - [\pi_A(B) + \pi_B(B)] &= \frac{2\alpha\Sigma k}{9}. \end{aligned}$$

Consequently,

$$s_A^{PP} = \frac{1}{2} + \frac{k - \Delta}{6}, \quad s_A^{NN} = \frac{1}{2} + \frac{k - \Delta}{6} + \frac{2\delta\Sigma k}{27},$$

and the price formulas give

$$p_A^{PP} - p_A^{NN} = \delta\alpha\Sigma \left(\frac{1}{3} + \frac{k}{27} \right), \quad p_B^{PP} - p_B^{NN} = \delta\alpha\Sigma \left(\frac{1}{3} - \frac{k}{27} \right).$$

Positive shares in the two no-DP groups imply $|k| + \Sigma < 3$, so both initial prices rise. Moreover, the profit and market-share expressions above imply

$$\frac{3\alpha}{2}(s_A^{PP} - s_A^{NN}) - \delta[\pi_B(B) - \pi_B(A)] = -\frac{\delta\alpha\Sigma}{3}.$$

Subtracting the expression for CS^d under NN from its value under PP , using this identity, and completing the square in the market-share terms gives (6), since $\alpha\Sigma = (v'_A - v_A) + (v'_B - v_B)$.

More generally, for uniform period-2 locations and positive shares, expected net utility per consumer in a group with net values $(\tilde{w}_A, \tilde{w}_B)$ is

$$u(\tilde{w}_A, \tilde{w}_B) = \frac{\tilde{w}_A + \tilde{w}_B}{2} - \frac{5\beta}{8} + \frac{[2(\tilde{w}_A - \tilde{w}_B) - (\beta_A - \beta_B)]^2}{72\beta}. \quad (16)$$

The derivative with respect to either net value lies between $1/3$ and $2/3$ when shares are interior. Thus, DP strictly increases expected consumer net utility in either initial group: $u^P > u(A)$ and

$u^P > u(B)$. Substituting these net utilities with $\beta_i = \alpha_i$ and the initial shares in (6) gives

$$\begin{aligned} \frac{CS^{PP} - CS^{NN}}{\delta\alpha} = & -\frac{\Sigma}{12} + \frac{7\Delta^2 - \Sigma^2}{72} + \frac{-18 + (7 - 12\delta)\Sigma}{324}k\Delta \\ & - \Sigma \left[\frac{7}{324} + \frac{5\delta\Sigma}{729} \right] k^2. \end{aligned} \quad (17)$$

Recall that $|k| + \Sigma < 3$, and hence $0 < \Sigma < 3$.

B.4 Proof of Proposition 4

We use (17), allowing unequal differentiation parameters in case (i).

B.4.1 Case (i): comparable gains from data.

Since $0 < \delta < 1$ and $\Sigma < 3$, $|-18 + (7 - 12\delta)\Sigma| \leq 18 + 5\Sigma$, and the absolute value of the coefficient of k^2 in (17) is at least $7\Sigma/324$. Completing the square to bound the final two terms therefore gives

$$\frac{CS^{PP} - CS^{NN}}{\delta\alpha\Sigma} \leq -\frac{1}{12} + \frac{[7(\Delta/\Sigma)^2 - 1]\Sigma}{72} + \frac{(\Delta/\Sigma)^2(18 + 5\Sigma)^2}{9072}.$$

The factor-of-three condition is equivalent to $|\Delta/\Sigma| \leq 1/2$. Using this and $\Sigma < 3$, the right-hand side is at most

$$-\frac{1}{12} + \frac{1}{32} + \frac{1089}{36288} = -\frac{89}{4032} < 0.$$

This proves case (i), including the extension to unequal differentiation parameters stated after Proposition 4.

B.4.2 Case (ii): sufficiently different gains from data.

Suppose $v'_B - v_B \geq 3(v'_A - v_A) + 3t$; the case with the firm labels reversed is identical. Since $\alpha = 2t$, this condition is equivalent to $\Delta \leq -(2\Sigma + 3)/4$. The strict NN conditions in Corollary 3 are $-(\Sigma + \Delta)/2 < k < (\Sigma - \Delta)/2$. Combining these with positive data gains and $|k| + \Sigma < 3$ gives

$$\frac{3}{2} < \Sigma < 3, \quad \Delta \leq -\frac{2\Sigma + 3}{4}, \quad -\frac{2\Sigma - 3}{8} < k < 3 - \Sigma.$$

In particular, the lower bound follows from $k > -(\Sigma + \Delta)/2 \geq -(2\Sigma - 3)/8$.

The right-hand side of (17) is concave in k . Its derivative with respect to δ is $-\Sigma k(27\Delta + 5\Sigma k)/729$, which has the sign of k on the displayed interval. It therefore suffices to bound the expression at $(k, \delta) = (-(2\Sigma - 3)/8, 1)$ and $(k, \delta) = (3 - \Sigma, 0)$; the boundary values of δ are used

only to obtain lower bounds. At both endpoints the expression decreases with Δ , so replacing Δ by $-(2\Sigma + 3)/4$ gives a further lower bound. The two endpoint expressions are then, respectively,

$$\begin{aligned}\frac{-80\Sigma^4 - 372\Sigma^3 + 1224\Sigma^2 - 1701\Sigma + 13122}{186624} &\geq \frac{31}{2304} > 0, \\ \frac{-112\Sigma^3 + 996\Sigma^2 - 2196\Sigma + 1863}{10368} &\geq \frac{1}{24} > 0.\end{aligned}$$

The first polynomial decreases on $[3/2, 3]$ and attains the stated bound at $\Sigma = 3$; the second increases on this interval and attains its bound at $\Sigma = 3/2$, as direct differentiation verifies. Thus $CS^{PP} > CS^{NN}$ for every $0 < \delta < 1$.

B.5 Ranking reversal in case (ii)

Under the assumptions of Proposition 4(ii), the bounds established in its proof imply

$$\begin{aligned}\frac{2(w_A - w_B)}{\alpha} &= k - \Delta > -\frac{\Sigma + 3\Delta}{2} > 0, \\ \frac{2(w'_A - w'_B)}{\alpha} &= k + \Delta < 3 - \Sigma + \Delta \leq \frac{9}{4} - \frac{3\Sigma}{2} < 0.\end{aligned}$$

Finally, the strict NN conditions mean that DP reduces joint second-period profit from either original customer group. With uniform locations, this is equivalent to making the firms' shares within each group more equal, as shown in Corollary 3.

C Personalized pricing

C.1 Forward-looking consumers

For a given consumer, write $w_i = v_i - c_i$, $w'_i = v'_i - c_i$, and $W_i = w_i + \delta w'_i$. Assume $\min\{w_A, w_B\} \geq 0$ and $w'_i > w_i$. These values can vary across consumers and are observed by both firms. Prices are personalized, but each firm's portability commitment covers all its customers. Consumers are forward looking and have a zero outside option in each period. First-period prices can be negative. All payoffs below are per consumer.

C.1.1 Proof of Proposition 5

In period 2, asymmetric Bertrand competition gives the sale to the firm creating the greater net value. The winner earns the difference in net values, and the consumer receives the lower net value. Equal net values give both firms zero profit, regardless of which firm serves the consumer.

Without DP, an initial purchase from A therefore gives

$$\pi_A(A) = \max\{w'_A - w_B, 0\}, \quad \pi_B(A) = \max\{w_B - w'_A, 0\}, \quad u(A) = \min\{w'_A, w_B\}.$$

The payoffs after an initial purchase from B follow by symmetry. If the initial supplier offers DP, the second-period payoffs instead become

$$\pi_A^P = \max\{w'_A - w'_B, 0\}, \quad \pi_B^P = \max\{w'_B - w'_A, 0\}, \quad u^P = \min\{w'_A, w'_B\}.$$

Consequently,

$$S(A) = \max\{w'_A, w_B\}, \quad S(B) = \max\{w_A, w'_B\}, \quad S^P = \max\{w'_A, w'_B\}. \quad (18)$$

In particular, $S^P \geq S(A)$ and $S^P \geq S(B)$.

Fix a portability profile d . Firm i 's lowest first-period margin consistent with preferring to win the consumer rather than lose is

$$p_i - c_i = \delta[\pi_i^d(j) - \pi_i^d(i)], \quad j \neq i.$$

Comparing the consumer's discounted net utilities at these margins gives A's relative advantage:

$$\Omega^d = w_A - w_B + \delta[S^d(A) - S^d(B)].$$

Here $S^d(i) = S^P$ if firm i offers DP and $S^d(i) = S(i)$ otherwise. The winning firm raises its initial price until the consumer is indifferent between its offer and the rival's best offer. Thus

$$\Pi_A^d = \max\{\Omega^d, 0\} + \delta\pi_A^d(B), \quad \Pi_B^d = \max\{-\Omega^d, 0\} + \delta\pi_B^d(A).$$

If $\Omega^d = 0$, either initial allocation, or any sharing of tied consumers, gives these same discounted profits. Total discounted surplus is also the same under either initial allocation, so discounted consumer surplus is unchanged.

Holding B's policy fixed, A's decision to offer DP replaces $S(A)$ with S^P and leaves $S^d(B)$ and $\pi_A^d(B)$ unchanged. Since $S^P \geq S(A)$, A's profit cannot fall. The symmetric argument applies to B.

To establish when A's preference is strict, first suppose $w'_A \geq w'_B$. Then $S^P = S(A)$, so A is indifferent. Now suppose $w'_B > w'_A$. In this case $S^d(B) = w'_B$ and $\pi_A^d(B) = 0$, regardless of B's policy. After A offers DP, $\Omega^d = w_A - w_B$. If $w_A \leq w_B$, A therefore earns zero with either decision.

If $w_A > w_B$, then $w'_A > w_A > w_B$, so A's profits without and with its own DP offer are respectively

$$\max\{w_A - w_B - \delta(w'_B - w'_A), 0\} \quad \text{and} \quad w_A - w_B.$$

Its gain is

$$\min\{w_A - w_B, \delta(w'_B - w'_A)\} > 0.$$

This proves A's strict-preference condition; B's follows by symmetry.

These comparisons hold for every consumer and either policy of the rival. Summing profits across consumers therefore establishes that offering DP is weakly preferred by each firm. If each firm has a positive measure of consumers satisfying its strict-preference condition, offering DP is strictly preferred by each firm, making bilateral DP the unique equilibrium.

C.1.2 Consumer surplus

Without DP, the profit formulas above imply

$$\Pi_A^{NN} = \max\{W_A - W_B, 0\}, \quad \Pi_B^{NN} = \max\{W_B - W_A, 0\}.$$

To verify this, if $w_A \leq w'_B$ and $w_B \leq w'_A$, then $S(A) = w'_A$, $S(B) = w'_B$, and $\pi_A(B) = \pi_B(A) = 0$. Hence $\Omega^{NN} = W_A - W_B$. If instead $w_A > w'_B$, A wins both periods, and substitution again gives $\Pi_A^{NN} = W_A - W_B$ and $\Pi_B^{NN} = 0$. The remaining case, $w_B > w'_A$, is symmetric.

The selected allocation generates total discounted surplus $\max\{W_A, W_B\}$. Subtracting profits therefore gives

$$CS^{NN} = \min\{W_A, W_B\}. \tag{19}$$

Under bilateral DP, both firms have access to data in period 2 regardless of the initial supplier, so competition separates across periods. Consequently,

$$\begin{aligned} CS^{PP} &= \min\{w_A, w_B\} + \delta \min\{w'_A, w'_B\} \\ &\leq \min\{w_A + \delta w'_A, w_B + \delta w'_B\} = CS^{NN}. \end{aligned} \tag{20}$$

The inequality is strict if and only if

$$(w_A - w_B)(w'_A - w'_B) < 0.$$

Thus bilateral DP weakly lowers consumer surplus for every consumer. Aggregate consumer surplus falls strictly whenever these ranking reversals occur on a set of positive measure.

C.2 Forward-looking consumers with possibly negative initial net values

Maintain the setting and notation of the preceding subsection, except that we now assume one or both initial net values to be negative. Specifically, assume $\min\{w_A, w_B\} < 0$ and impose

$$\min\{w'_A, w'_B\} > 0, \quad \max\{W_A, W_B\} > 0. \quad (21)$$

The first condition ensures that each firm generates positive net value when it has access to the consumer's data. If firm i has $w'_i < 0$, then it is not relevant for the consumer in either period, even if it benefits from portability, and so the other firm would hold a monopoly position, which makes the choice of portability irrelevant for the consumer. The second condition ensures that at least one firm generates strictly positive discounted net value by serving the consumer in both periods. Otherwise, neither firm would ever find it profitable to attract the consumer in any period.

We also use the following convention under every portability profile: at $W_i = 0$, we select the equilibrium payoffs obtained as W_i approaches zero from above. Consumers purchase when indifferent between purchasing and not purchasing.

We prove that each firm still weakly prefers to offer DP, whether or not the rival offers it. The argument requires several lemmas.

Lemma 2. *If $w_A \geq w'_B$, then $\Pi_A = W_A - \max\{W_B, 0\}$ and $\Pi_B = 0$, regardless of the DP regime. And if $w_B \geq w'_A$, then $\Pi_A = 0$ and $\Pi_B = W_B - \max\{W_A, 0\}$, regardless of the DP regime.*

Proof of Lemma 2 Suppose $w_A \geq w'_B$. This means A wins period 2 no matter the outcome in period 1, so we have $u(A) = \max\{w_B, 0\}$ and $u(B) = u^P = w'_B$.

Since B never wins in period 2, it is unwilling to subsidize in period 1, so the highest present discounted value (PDV) of net utility that B can offer the consumer in period 1 is $w_B + \delta w'_B$, by setting $p_B = c_B$. The outside option for the consumer in period 1 is to join neither firm, in which case she will end up joining firm A in period 2, obtaining a PDV of net utility $\delta \max\{w_B, 0\}$. Note that $w'_B > 0$ implies that

$$\max\{w_B + \delta w'_B, \delta \max\{w_B, 0\}\} = \max\{w_B + \delta w'_B, 0\} = \max\{W_B, 0\}.$$

Thus, to win the consumer in period 1, A sets p_A such that

$$w_A - (p_A - c_A) + \delta u^d(A) = \max\{W_B, 0\},$$

leading to profits

$$\begin{aligned}
\Pi_A &= (p_A - c_A) + \delta[w'_A - u^d(A)] \\
&= w_A + \delta u^d(A) - \max\{W_B, 0\} + \delta[w'_A - u^d(A)] \\
&= W_A - \max\{W_B, 0\} > 0.
\end{aligned}$$

Since this is positive, A wins both periods and $\Pi_B = 0$.

The result for the case $w_B \geq w'_A$ follows by symmetry in A and B.

■

For the remainder of this subsection, we focus on the case $w'_A > w_B$ and $w'_B > w_A$. This means that if a firm does not offer DP and wins in period 1, it will also win in period 2.

We first determine the equilibrium for the case when neither firm has committed to DP.

Lemma 3. *Without DP, the equilibrium outcome is as follows. If $W_A > W_B$, then firm A attracts the consumer in both periods, resulting in profits*

$$\Pi_A = W_A - \max\{W_B, 0\} \quad \text{and} \quad \Pi_B = 0.$$

The results for the case when $W_B > W_A$ are obtained by symmetry in A and B. If $W_A = W_B > 0$, either firm may serve the consumer in both periods; both firms earn zero and consumer surplus is W_A .

Proof of Lemma 3. If $W_B \leq 0$, then B cannot exert any competitive pressure on A in either period, so the binding constraint is the outside option, meaning that $\Pi_A = W_A$. To see this, note that if B does not win the consumer in the first period, then B is irrelevant in the second period because $w_B < 0$. This implies that the lowest price p_B that B is willing to offer in the first period is such that $p_B - c_B = -\delta(w'_B - \max\{w_A, 0\})$, which means the highest PDV of net utility that B can offer in the first period is

$$w_B + \delta(w'_B - \max\{w_A, 0\}) + \delta \max\{w_A, 0\} = W_B \leq 0.$$

Thus, B is also irrelevant in the first period.

Suppose then $W_A > W_B > 0$. First, we show that in equilibrium the consumer must join one of the two firms in period 1. Suppose she doesn't. Then her PDV of net utility is

$$\delta \max\{\min\{w_A, w_B\}, 0\} = 0$$

and firm A earns discounted profit

$$\delta \max \{w_A - \max \{w_B, 0\}, 0\} = \delta \max \{w_A, 0\},$$

where both equalities follow from $\min \{w_A, w_B\} < 0$. But firm A could attract the consumer in period 1 (and therefore also in period 2) by setting p_A such that

$$w_A - (p_A - c_A) + \delta \max \{w_B, 0\} = 0,$$

leading to total profits

$$p_A - c_A + \delta(w'_A - \max \{w_B, 0\}) = W_A > \delta \max \{w_A, 0\}.$$

Second, the consumer cannot join firm B in equilibrium. If she did, firm B attracts her in both periods, so her net utility from the perspective of period 1 would be $w_B - (p_B - c_B) + \delta \max \{w_A, 0\}$. Firm A's profits are zero and firm B's profits are $p_B - c_B + \delta(w'_B - \max \{w_A, 0\})$, which must be non-negative. This implies

$$w_B - (p_B - c_B) + \delta \max \{w_A, 0\} \leq W_B.$$

But then firm A could attract the consumer in period 1 (and therefore also in period 2) by choosing some $0 < \varepsilon < W_A - W_B$ and setting p_A such that

$$w_A - (p_A - c_A) + \delta \max \{w_B, 0\} = W_B + \varepsilon,$$

leading to strictly positive total profit

$$p_A - c_A + \delta(w'_A - \max \{w_B, 0\}) = W_A - W_B - \varepsilon > 0.$$

Thus, A must win the consumer in both periods and its price p_A in the first period must be such that

$$w_A - (p_A - c_A) + \delta \max \{w_B, 0\} = \max \{W_B, 0\} = W_B.$$

This means A's profit is $\Pi_A = W_A - W_B$, while consumer surplus is W_B .

When $W_A = W_B > 0$, competition gives the consumer this common value. Either firm can serve both periods, and each firm's discounted profit is zero.

■

Now consider the case when both firms have committed to DP (or DP has been imposed by

policy).

Lemma 4. *Suppose both firms have committed to DP. If $w'_A \geq w'_B$ and $(W_B < 0$ or $w_A \geq w_B)$, then A wins both periods, resulting in profits*

$$\Pi_A^{PP} = W_A - \max\{W_B, 0\} \quad \text{and} \quad \Pi_B^{PP} = 0.$$

If $w'_A \geq w'_B$ and $W_B \geq 0$ and $w_B > w_A$, then B wins the first period and A wins the second period, resulting in profits

$$\Pi_A^{PP} = \delta(w'_A - w'_B) \quad \text{and} \quad \Pi_B^{PP} = \min\{w_B - w_A, W_B\}.$$

The results for the case $w'_A \leq w'_B$ are obtained by symmetry in A and B.

Proof of Lemma 4 First consider $W_A \neq 0$ and $W_B \neq 0$; the maintained selection convention gives the payoffs at either zero boundary by taking the limit from above. Suppose both firms have committed to DP, and recall $w'_A > w_B$, $w'_B > w_A$. In equilibrium, the consumer must buy from one of the two firms in period 1. If not, then she would obtain PDV of net utility $\delta \max\{\min\{w_A, w_B\}, 0\} = 0$ and firm $i \in \{A, B\}$ such that $W_i > 0$ (recall this must be true for $i = A$ or $i = B$) could profitably deviate by setting its first-period price p_i such that

$$w_i - (p_i - c_i) + \delta \min\{w'_A, w'_B\} = 0,$$

yielding profits

$$p_i - c_i + \delta \max\{w'_i - w'_j, 0\} = W_i > \delta \max\{w_i, 0\}.$$

The right-hand side is firm i 's discounted profit if the consumer does not buy initially given that $\min\{w_A, w_B\} < 0$.

Conditional on an initial purchase being made, the period-2 outcome does not depend on the initial supplier. Thus neither firm is willing to subsidize the initial purchase merely to win it from its rival. A subsidy may nevertheless be profitable when the alternative is no initial purchase.

Suppose $w'_A \geq w'_B$, so A wins in period 2 regardless of who wins in period 1. If A also wins in period 1, then we must have

$$w_A - (p_A - c_A) + \delta w'_B = \max\{W_B, \delta \max\{\min\{w_A, w_B\}, 0\}\} = \max\{W_B, 0\}.$$

This implies

$$\Pi_A^{PP} = W_A - \max\{W_B, 0\} \quad \text{and} \quad \Pi_B^{PP} = 0,$$

and this solution is valid only if $W_A \geq \max\{W_B, 0\}$ and A does not want to deviate by slightly

raising p_A . This deviation can only be profitable if $W_B \geq 0$, in which case the deviation yields profits of $\delta(w'_A - w'_B)$. Thus, the deviation is not profitable if $W_B < 0$ or $w_A - w_B \geq 0$. So this solution is valid if and only if $W_B < 0$ or $w_A - w_B \geq 0$ (note that either inequality implies $W_A \geq \max\{W_B, 0\}$ because we are in the case $w'_A \geq w'_B$).

If B wins period 1, then we must have

$$w_B - (p_B - c_B) + \delta w'_B = \max\{w_A + \delta w'_B, 0\},$$

so B's profits are $W_B - \max\{w_A + \delta w'_B, 0\}$. This solution is therefore valid if and only if (iff) $W_B \geq 0$ and $w_B - w_A \geq 0$, in which case

$$\Pi_A^{PP} = \delta(w'_A - w'_B) \quad \text{and} \quad \Pi_B^{PP} = \min\{w_B - w_A, W_B\}.$$

■

Finally, suppose only one firm (A) has committed to DP.

Lemma 5. *Suppose firm A has committed to DP, while firm B has not. If $w'_A \geq w'_B$, discounted profits and consumer surplus are the same as without DP, so*

$$\Pi_A^{PN} = \max\{W_A - \max\{W_B, 0\}, 0\} \quad \text{and} \quad \Pi_B^{PN} = \max\{W_B - \max\{W_A, 0\}, 0\}.$$

If $w'_B > w'_A$, then discounted profits and consumer surplus are the same as under bilateral DP, so

$$\Pi_A^{PN} = 0 \quad \text{and} \quad \Pi_B^{PN} = W_B - \max\{W_A, 0\}$$

when $W_A < 0$ or $w_B - w_A \geq 0$, and

$$\Pi_A^{PN} = \min\{w_A - w_B, W_A\} \quad \text{and} \quad \Pi_B^{PN} = \delta(w'_B - w'_A)$$

when $W_A \geq 0$ and $w_A - w_B > 0$.

Proof of Lemma 5 Suppose firm A has committed to DP, while firm B has not. In equilibrium, the consumer buys from one of the two firms in period 1. If not, then she would obtain PDV of net utility zero. If $W_A > 0$, firm A could profitably deviate by setting its first-period price p_A such that

$$w_A - (p_A - c_A) + \delta \min\{w'_A, w'_B\} = 0,$$

yielding profits

$$p_A - c_A + \delta \max\{w'_A - w'_B, 0\} = W_A > \delta \max\{w_A, 0\}.$$

The right-hand side is A's discounted profit when the consumer has not made an initial purchase. If $W_A \leq 0$, then $W_B > 0$. Firm B can instead attract the consumer by setting

$$p_B - c_B = w_B + \delta \max\{w_A, 0\}.$$

This gives the consumer zero discounted net utility and B discounted profit $W_B > \delta \max\{w_B, 0\}$.

If $w'_A \geq w'_B$, then whichever firm wins period 1 also wins period 2. In this case, the analysis is the same as with no DP, leading to

$$\Pi_A^{PN} = \max\{W_A - \max\{W_B, 0\}, 0\} \quad \text{and} \quad \Pi_B^{PN} = \max\{W_B - \max\{W_A, 0\}, 0\}.$$

If $w'_B > w'_A$, then firm B wins period 2, regardless of who wins period 1. In this case, the analysis is the same as with bilateral DP, so profits and consumer surplus are the same as in Lemma 4 for the case $w'_B > w'_A$.

■

C.2.1 Incentives to offer portability

Start with no DP by either firm and suppose $W_A \geq \max\{W_B, 0\}$, so we may take A to win in the absence of DP, with profits

$$\Pi_A = W_A - \max\{W_B, 0\} \quad \text{and} \quad \Pi_B = 0.$$

Then, from Lemma 5, if $w'_A \geq w'_B$, discounted profits and consumer surplus with unilateral DP by A are the same as without DP.

If $w'_A < w'_B$ then we must have $w_A > w_B$, so profits are

$$\Pi_A^{PN} = \min\{w_A - w_B, W_A\} \quad \text{and} \quad \Pi_B^{PN} = \delta(w'_B - w'_A)$$

It is easily seen that B's profits are strictly higher than without DP. As for A, if $W_B \leq 0$, then

$$w_B + \delta w'_A < w_B + \delta w'_B \leq 0,$$

so A's profits are W_A in both cases. If $W_B > 0$, then A's profits are strictly higher with unilateral DP by A because

$$\min\{w_A - w_B, W_A\} > W_A - W_B.$$

Thus, A has an incentive to offer unilateral DP, and strictly so provided $w'_A < w'_B$ and $W_B > 0$.

Now compare the profits under unilateral DP by A (Lemma 5) to the profits under bilateral DP (Lemma 4). If $w'_B > w'_A$, then discounted profits and consumer surplus are the same under unilateral DP by A and bilateral DP.

If $w'_A \geq w'_B$, then profits under unilateral DP by A are

$$\Pi_A^{PN} = W_A - \max\{W_B, 0\} \quad \text{and} \quad \Pi_B^{PN} = 0.$$

If $w'_A \geq w'_B$ and $(W_B < 0 \text{ or } w_A \geq w_B)$, these are also the profits under bilateral DP.

If $w'_A \geq w'_B$ and $W_B \geq 0$ and $w_B \geq w_A$, then profits under bilateral DP are

$$\Pi_A^{PP} = \delta(w'_A - w'_B) \quad \text{and} \quad \Pi_B^{PP} = \min\{w_B - w_A, W_B\}.$$

Clearly, B's profits are higher, strictly so if $w'_A \geq w'_B$ and $W_B > 0$ and $w_B > w_A$. And A's profits are also higher in this case, because $\delta(w'_A - w'_B) \geq W_A - W_B$.

Lemma 5 and its symmetric counterpart imply that each firm's gain from offering DP is the same under either policy of its rival. Hence the comparisons above also establish offering incentives against the rival's other policy.

In summary, firm A does at least as well and in some cases strictly better by deviating to unilateral DP from the situation without any DP. And firm B does at least as well and in some cases strictly better by deviating from the case with unilateral DP by A to the case with bilateral DP. Thus, bilateral DP is an equilibrium. It is the unique equilibrium if there is a positive measure of consumers satisfying

$$w_A > w_B, \quad w'_B > w'_A, \quad \min\{W_A, W_B\} > 0,$$

and a positive measure satisfying

$$w_B > w_A, \quad w'_A > w'_B, \quad \min\{W_A, W_B\} > 0.$$

The result for the case $W_B \geq \max\{W_A, 0\}$ follows by symmetry in A and B.

C.3 Myopic consumers

Consider a consumer configuration (v_A, v_B, v'_A, v'_B) , with $w_i = v_i - c_i$ and $w'_i = v'_i - c_i > w_i$ for $i \in \{A, B\}$. Prices are fully personalized, consumers maximize current-period surplus, and firms discount future profits by $\delta \in (0, 1)$. We first assume $w_A, w_B \geq 0$ (Section C.3.4 allows both to be negative). Write NN , PN , NP and PP for the four portability profiles, with A's policy listed first. And Π_i^{NN} , Π_i^{PN} , Π_i^{NP} and Π_i^{PP} denote firm i 's discounted profits under these respective

profiles.

C.3.1 Individual incentives

The analysis follows the forward-looking case, except that consumers ignore their expected second-period surplus when choosing their initial supplier. Thus, when neither firm offers portability, firm A earns

$$\begin{aligned}\Pi_A^{NN} = \max\{w_A - w_B + \delta[\pi_A(A) + \pi_B(A) - \pi_A(B) - \pi_B(B)], 0\} \\ + \delta\pi_A(B).\end{aligned}$$

If A offers portability while B withholds it, its profit becomes

$$\begin{aligned}\Pi_A^{PN} = \max\{w_A - w_B + \delta[\pi_A^P + \pi_B^P - \pi_A(B) - \pi_B(B)], 0\} \\ + \delta\pi_A(B).\end{aligned}$$

Comparing these expressions, A weakly prefers offering portability if and only if

$$\pi_A^P + \pi_B^P \geq \pi_A(A) + \pi_B(A), \quad (22)$$

provided at least one of the two maxima is positive.³ The same comparison applies when B offers portability: changing A 's policy replaces $\pi_A(A) + \pi_B(A)$ by $\pi_A^P + \pi_B^P$ inside its first-period profit term, leaving its continuation profit after an initial purchase from B unchanged.

Second-period Bertrand competition gives

$$\pi_A(A) + \pi_B(A) = |w'_A - w_B|, \quad \pi_A^P + \pi_B^P = |w'_A - w'_B|.$$

Hence A 's joint-profit condition is

$$|w'_A - w'_B| \geq |w'_A - w_B|.$$

Symmetrically, B 's condition is

$$\pi_A^P + \pi_B^P \geq \pi_A(B) + \pi_B(B), \quad \text{or equivalently} \quad |w'_A - w'_B| \geq |w_A - w'_B|.$$

The relevant comparison therefore concerns joint second-period profits, rather than total surplus as with forward-looking consumers.

³If both maxima are zero, A is indifferent even when the inequality fails. Thus the condition is always sufficient for a weak preference, and necessary when A strictly wins the initial sale under at least one policy. The same qualification applies to the other comparisons below.

C.3.2 Equilibria for a common consumer configuration

Proposition 7. *Suppose all consumers share a configuration with $w_A \geq w_B \geq 0$. If $w'_A \geq w'_B$ and at least one ranking is strict, the pure-strategy equilibria are exactly NN and NP : A strictly prefers withholding portability, and B is indifferent given that A withholds.*

If $w'_B \geq w'_A$, including ties, the pure-strategy equilibria are characterized by

Profile	Necessary and sufficient condition
NN	$2w'_A \geq w_B + w'_B \quad \text{or} \quad w_A - w_B \leq \delta(w'_A - w_A)$
PN	$w'_B - w'_A \geq \min \left\{ w'_A - w_B, \frac{1 + \delta}{\delta}(w_A - w_B) \right\}$
NP	$w'_B - w'_A \leq \min \left\{ w'_A - w_B, \frac{1 + \delta}{\delta}(w_A - w_B) \right\}$
PP	$2w'_A \leq w_B + w'_B \quad \text{and} \quad w_A - w_B \geq \delta(w'_A - w_A).$

In both cases, B weakly prefers withholding portability against either policy of A . Whenever PP is an equilibrium, PN is also an equilibrium. The results are symmetric when $w_B \geq w_A$.

Proof. Suppose first that $w_A \geq w_B$ and $w'_A \geq w'_B$, with at least one inequality strict. When A withholds portability, it strictly wins the initial sale against either policy of B , since the respective expressions inside its profit maxima are

$$\begin{aligned} w_A - w_B + \delta(w'_A - w_B - |w_A - w'_B|) &> 0, \\ w_A - w_B + \delta(w'_B - w_B) &> 0. \end{aligned}$$

For the first inequality, if $w_A \geq w'_B$, the expression in parentheses is $(w'_A - w_A) + (w'_B - w_B) > 0$; otherwise it is $(w'_A - w'_B) + (w_A - w_B)$. Portability reduces joint continuation profits from $w'_A - w_B$ to $w'_A - w'_B$, so A strictly prefers withholding it. Firm B earns zero under both NN and NP , which are therefore exactly the equilibria.

Now suppose $w_A \geq w_B$ and $w'_B \geq w'_A$. Substituting the second-period profits into the first-period profit expressions gives

$$\begin{aligned} \Pi_A^{NN} &= \max\{(1 + \delta)(w_A - w_B) - \delta(w'_B - w'_A), 0\}, \\ \Pi_A^{PN} &= \max\{w_A - w_B - \delta(w'_A - w_A), 0\}, \\ \Pi_A^{NP} &= \max\{w_A - w_B + \delta(2w'_A - w_B - w'_B), 0\}, \\ \Pi_A^{PP} &= w_A - w_B, \end{aligned}$$

and

$$\begin{aligned}\Pi_B^{NN} &= \max\{(1 + \delta)(w_B - w_A) + \delta(w'_B - w'_A), 0\}, \\ \Pi_B^{PN} &= \max\{w_B - w_A + \delta(w'_A - w_A), 0\} + \delta(w'_B - w'_A), \\ \Pi_B^{NP} &= \max\{w_B - w_A - \delta(2w'_A - w_B - w'_B), 0\}, \\ \Pi_B^{PP} &= \delta(w'_B - w'_A).\end{aligned}$$

These expressions imply $\Pi_B^{NN} \geq \Pi_B^{NP}$ and $\Pi_B^{PN} \geq \Pi_B^{PP}$, so B weakly prefers withholding portability.

For NN , only A 's constraint matters: $\Pi_A^{NN} \geq \Pi_A^{PN}$ holds exactly when $2w'_A \geq w_B + w'_B$ or $w_A - w_B \leq \delta(w'_A - w_A)$. Reversing this comparison gives the condition for PN in the table. For NP , B is willing to offer portability exactly when

$$(1 + \delta)(w_A - w_B) \geq \delta(w'_B - w'_A).$$

Under this condition, A is willing to withhold portability exactly when $2w'_A \geq w_B + w'_B$, giving the NP row. For PP , B is willing to offer portability exactly when $w_A - w_B \geq \delta(w'_A - w_A) > 0$; A is then willing to offer portability exactly when $2w'_A \leq w_B + w'_B$. These conditions also imply the PN condition. $\square$

C.3.3 Consumer surplus and a portability mandate

A mandate can benefit myopic consumers. Consider a configuration satisfying

$$0 < w'_B - w'_A < w_A - w_B.$$

Since $w'_A > w_A$, this implies $w'_B - w'_A < w'_A - w_B$, so NN is a voluntary equilibrium by Proposition 7. Under NN , A wins in both periods: its first-period advantage satisfies

$$(1 + \delta)(w_A - w_B) - \delta(w'_B - w'_A) > 0.$$

Its discounted profit equals this expression, while B earns zero. Subtracting profits from total surplus $w_A + \delta w'_A$ yields

$$CS^{NN} = w_B + \delta(w'_B - w_A) + \delta w_B.$$

A mandate imposing PP makes A the first-period supplier and B the second-period supplier. Their respective profits are $w_A - w_B$ and $\delta(w'_B - w'_A)$, so

$$CS^{PP} = w_B + \delta w'_A.$$

Consequently,

$$CS^{PP} - CS^{NN} = \delta[(w_A - w_B) - (w'_B - w'_A)] > 0.$$

C.3.4 Negative initial net values and total market failure

Proposition 8. *Suppose consumers are myopic and have the same configuration of values, which satisfy*

$$\min\{w_A + \delta w'_A, w_B + \delta w'_B\} > 0, \quad \max\{w_A, w_B\} < 0.$$

If

$$\max\{w_A, w_B\} + \delta|w'_A - w'_B| < 0,$$

then, relative to NN, a portability mandate eliminates trade in both periods and reduces consumer surplus from $\min\{w_A + \delta w'_A, w_B + \delta w'_B\} > 0$ to zero. Both firms earn zero profit under the mandate; without portability, the firm with the larger discounted net value earns the difference, while the other earns zero.

Proof. Without portability, the initial supplier is a monopolist in period 2: its data-enabled net value is positive, while its rival's net value remains negative. Its second-period profit is w'_i , so its maximum profitable introductory offer gives the consumer surplus $w_i + \delta w'_i > 0$. First-period competition therefore generates trade, leaves the consumer $\min\{w_A + \delta w'_A, w_B + \delta w'_B\}$, and gives the winning firm the difference between the two discounted net values. The initial supplier also serves the consumer in period 2.

Under a mandate, attracting a myopic consumer initially requires a price margin no greater than $w_i < 0$. The firm's second-period profit is at most $|w'_A - w'_B|$, so its discounted profit from initiating trade is at most

$$w_i + \delta|w'_A - w'_B| \leq \max\{w_A, w_B\} + \delta|w'_A - w'_B| < 0.$$

Neither firm initiates trade, no data are generated, and negative initial net values also preclude trade in period 2. Profits and consumer surplus are therefore zero.

Finally, *NN* is a voluntary equilibrium: a unilateral portability provider cannot profitably initiate trade by the same inequality, and cannot serve the rival's customers when the rival withholds their data. Such a deviation yields zero profit and hence cannot improve its payoff. $\square$

D Imperfect data portability

D.1 General incentives to offer portability

The profit formulas in Lemma 1 continue to apply with partial portability:

$$\Omega^d = w_A - w_B + \delta[S^d(A) - S^d(B)],$$

$$\Pi_A^d = H_A(\Omega^d) + \delta\pi_A^d(B), \quad \Pi_B^d = H_B(\Omega^d) + \delta\pi_B^d(A).$$

Here $S^d(i) = S(\theta; i)$ if firm i offers DP and $S^d(i) = S(0; i)$ otherwise; $\pi_j^d(i)$ is defined analogously using $\pi_j(\theta; i)$ and $\pi_j(0; i)$. Fix the rival's portability decision. When A changes from withholding to offering DP, Ω^d changes by

$$\delta[S(\theta; A) - S(0; A)],$$

while the final term $\delta\pi_A^d(B)$ in A's profit remains unchanged. Similarly, B's decision to offer DP changes Ω^d by $-\delta[S(\theta; B) - S(0; B)]$, while $\delta\pi_B^d(A)$ remains unchanged. Positive first-period shares ensure that H_A is strictly increasing and H_B is strictly decreasing. Thus firm i weakly prefers to offer DP if and only if $S(\theta; i) \geq S(0; i)$, with strict inequality giving a strict preference and equality giving indifference.

With myopic consumers, the same argument applies to $\tilde{\Omega}^d$, replacing each total-surplus expression by the corresponding joint second-period profit, as in Proposition 3. Firm i therefore weakly prefers to offer DP if and only if

$$\pi_A(\theta; i) + \pi_B(\theta; i) \geq \pi_A(0; i) + \pi_B(0; i).$$

Neither comparison depends on the rival's portability decision.

D.2 Proof of Proposition 6

Use $\beta = \beta_A + \beta_B$ as defined at the start of the Online Appendix. For uniform G and period-2 net values $(\tilde{w}_A, \tilde{w}_B)$, total surplus is given by (12), and joint profit per consumer is given by (15). When A offers DP, the net values for its original customers change from (w'_A, w_B) to $(w'_A, w_B + \theta(v'_B - v_B))$. Subtracting their total surplus without DP from that with DP gives

$$\begin{aligned} & S(\theta; A) - S(0; A) \\ &= \frac{5\theta(v'_B - v_B)}{9\beta} \left[\frac{7\beta_A + 2\beta_B}{5} - w'_A + w_B + \frac{\theta}{2}(v'_B - v_B) \right]. \end{aligned}$$

The corresponding change in joint second-period profit is

$$\begin{aligned} & \pi_A(\theta; A) + \pi_B(\theta; A) - \pi_A(0; A) - \pi_B(0; A) \\ &= \frac{2\theta(v'_B - v_B)}{9\beta} [\beta_A - \beta_B - 2(w'_A - w_B) + \theta(v'_B - v_B)]. \end{aligned}$$

Similarly, B's DP changes the net values for its original customers from (w_A, w'_B) to $(w_A + \theta(v'_A - v_A), w'_B)$. The change in total surplus is

$$\begin{aligned} & S(\theta; B) - S(0; B) \\ &= \frac{5\theta(v'_A - v_A)}{9\beta} \left[\frac{2\beta_A + 7\beta_B}{5} - w'_B + w_A + \frac{\theta}{2}(v'_A - v_A) \right], \end{aligned}$$

and the change in joint second-period profit is

$$\begin{aligned} & \pi_A(\theta; B) + \pi_B(\theta; B) - \pi_A(0; B) - \pi_B(0; B) \\ &= \frac{2\theta(v'_A - v_A)}{9\beta} [\beta_B - \beta_A - 2(w'_B - w_A) + \theta(v'_A - v_A)]. \end{aligned}$$

Since $v'_i - v_i > 0$, $\beta > 0$, and $\theta > 0$, the factors outside the brackets are strictly positive. The bracket signs establish the four conditions in the proposition, including strict preferences and indifference. Each bracket is strictly increasing in θ , which establishes the threshold property. The four changes are strictly convex quadratics in θ : if the coefficient of θ is negative, the relevant change initially falls below zero and returns to zero at a positive threshold. Hence the gain need not increase at every effectiveness level. At $\theta = 0$, offering DP leaves all outcomes unchanged.

Independence from the rival's decision gives the equilibrium statements for forward-looking consumers. With myopic consumers, adding the two weak offering conditions would require

$$(v'_A - v_A) + (v'_B - v_B) \leq \frac{\theta}{2} [(v'_A - v_A) + (v'_B - v_B)].$$

This is impossible for $0 < \theta \leq 1$, because both improvements are positive. Thus bilateral DP is never an equilibrium with myopic consumers.

D.3 Additional results and example

For each firm, the forward-looking bound exceeds the myopic bound by $9\beta/10 > 0$. Any firm weakly willing to offer DP with myopic consumers therefore strictly prefers to offer it with forward-looking consumers.

To verify the claim about second-period consumer surplus, use the net utility formula (16). Its

derivatives with respect to the two net values satisfy

$$\frac{\partial u}{\partial \tilde{w}_A} = \frac{1 + \tilde{s}_A}{3} > 0, \quad \frac{\partial u}{\partial \tilde{w}_B} = \frac{2 - \tilde{s}_A}{3} > 0,$$

where $\tilde{s}_A$ is A's share of the group. Increasing θ raises only the recipient's net value, so it strictly increases expected second-period consumer surplus in that group. The uniform share formula also shows that the recipient's share increases.

The positive-share conditions without DP are

$$-\beta_A - 2\beta_B < w_A - w'_B < w'_A - w_B < 2\beta_A + \beta_B.$$

For every $\theta \in [0, 1]$, the two net-value differences $w'_A - w_B - \theta(v'_B - v_B)$ and $w_A + \theta(v'_A - v_A) - w'_B$ lie between these two no-portability differences. Thus positive period-2 shares without DP ensure positive shares at every effectiveness level. Market coverage and positive first-period shares remain separate maintained assumptions.

Finally, in the example with $\alpha_A = \alpha_B = \beta_A = \beta_B = t$, $w_A = w_B$, and both data improvements equal to $12t/5$, each forward-looking condition is $9t/5 \geq (12t/5)(1 - \theta/2)$, or $\theta \geq 1/2$. Both no-DP net-value differences have absolute value $12t/5 < 3t$, so period-2 shares are positive. To verify first-period shares for this example, it suffices to choose uniform F . The total-surplus change for either original customer group is $2t\theta(2\theta - 1)/5$, whose absolute value is at most $2t/5$. Since $w_A = w_B$, it follows that $|\Omega^d| \leq 2\delta t/5 < 3t$ under every portability outcome. The common level of net values can be chosen sufficiently high to ensure coverage. The example is therefore consistent with all maintained assumptions. Setting $\theta = 1$ gives the conditions in Corollaries 1 and 3; continuity gives robustness of strict offering preferences to small reductions in effectiveness.