Online Appendix: Platform Traps

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A Infinite period model

In this section, we consider a multi-period setting to illustrate how our single-period model can be interpreted as capturing the full decision-making process across periods. We had claimed in the discussion of our model that the absence of discounting across the sellers' decisions within a period reflects the assumption that joining decisions are made over a short time frame, relative to the longer horizon (possibly infinite) over which payoffs arise. We illustrate this here by considering an infinite period model to show that a platform trap can still arise in such a setting.

Let's consider the general setting of Section 5, of which the marketplace baseline setting is a special case (i.e., the sellers' payoff functions can be mapped to a special case of the agents' payoff functions). The platform sets a single price in each period that applies to all participating agents in that period. In each of the first N periods, each agent is selected in sequence to decide whether to participate on the platform or not, so one agent decides in each period. If some agents have previously joined the platform, in any subsequent period they can leave, but are assumed to stay whenever staying is consistent with an equilibrium given the price the platform sets in that period. If indifferent between joining or not, an agent will join. Agents only get one chance to participate, and this only happens in the first N periods. This means if they decide not to participate (or if they later leave the platform), they remain forever outside. The discount factor is $0 < \delta < 1$.

We consider the two extreme cases considered in Section 5, corresponding to Corollary 1 and Corollary 2.

A.1 No pivotal agents

Let's first assume $\Delta(1) > 0$ which implies no agents are ever pivotal. We solve by backwards induction. Suppose at the start of period $N+1$, after which no more agents can join, $0 \leq n \leq N$ agents have joined. They obtain a benefit of being on the platform of $b(n)$ in each period, provided they all remain. The option for an individual agent at that point is to leave and obtain the alternative $u(n-1)$ in all subsequent periods, assuming the rest of the agents remain on the platform. Therefore the highest price the platform can set each period such that it is an equilibrium for all agents to remain (when they expect others to do so) is $P^{N+1}(n) = \Delta(n) = b(n) - u(n-1) > 0$. Since this logic applies in all subsequent periods, we also have $P^{t}(n) = \Delta(n)$ for all $t \geq N+1$.

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Agents will therefore get $u(n-1)$ in period $N+1$ and each subsequent period, implying each participating agent's present discounted value (PDV) payoff from the perspective of period $N+1$ is

$$V^{N+1}(n) = \frac{1}{1-\delta} u(n-1),$$

while the platform will obtain a PDV payoff of

$$\Pi^{N+1}(n) = \frac{1}{1-\delta} n \Delta(n) > 0.$$

Now consider the last period N in which agents can decide to join, which means agent N is deciding whether to join and some n satisfying $0 \leq n \leq N-1$ agents who have already joined the platform decide whether to stay. If agent N joins and the other n agents remain, each of the $n+1$ participating agents obtains a payoff equal to

$$b(n+1) - P + \delta V^{N+1}(n+1),$$

where P is the price charged by the platform. This is an equilibrium iff agent N indeed wants to join and none of the n agents that have previously joined want to leave in period N. The payoff to such a deviation is $u(n)$ in each subsequent period, so agent N joins and the existing n agents stay iff

$$b(n+1) - P + \delta V^{N+1}(n+1) \geq \frac{1}{1-\delta} u(n).$$

Thus, the maximum price the platform can set to induce participation of these $n+1$ agents is

$$P^N(n) = \Delta(n+1) > 0. \tag{9}$$

If the platform sets a price greater than $\Delta(n+1)$, then the only equilibrium is that no agents participate in period N. To see this, suppose $k \geq 1$ agents participate in equilibrium, which implies we must have

$$b(k) - \Delta(n+1) + \delta V^{N+1}(k) > \frac{1}{1-\delta} u(k-1),$$

i.e.,

$$b(k) + \delta V^{N+1}(k) - \frac{1}{1-\delta} u(k-1) > \Delta(n+1).$$

Recall $V^{N+1}(k) = \frac{1}{1-\delta} u(k-1)$, so the LHS is equal to $\Delta(k)$, which is increasing in k. Thus, since $k \leq n+1$, we must have

$$\Delta(n+1) \geq \Delta(k) > \Delta(n+1),$$

which is a contradiction. Thus, when the platform charges a price higher than $P^N(n) = \Delta(n+1)$, the only possible equilibrium is that no agents participate. This means the platform's optimal price in period N is indeed $P^N(n) = \Delta(n+1) > 0$.

The platform attracts agent N at the price (9) and each participating agent's present-discounted value (PDV) payoff in period N is

$$V^N(n) = \frac{u(n)}{1-\delta}.$$

The PDV of platform profit is

$$\begin{aligned} \Pi^N(n) &= (n+1)P^N(n) + \delta\Pi^{N+1}(n+1) \\ &= \frac{1}{1-\delta}(n+1)\Delta(n+1) > 0. \end{aligned}$$

so the platform will indeed attract agent N regardless of the value of n.

The induction hypothesis for $T+1 \in \{2, \ldots, N\}$ is as follows. For any $t \geq T+1$ and $n \leq t-1$, if n agents have already joined the platform at the beginning of period t, the platform induces all of them to remain and agent t to join by setting $P^t(n) = \Delta(n+1)$ if $t \leq N$ or $P^t(n) = \Delta(n)$ if $t \geq N+1$. All participating agents obtain PDV of payoffs equal to $V^t(n)$, where

$$V^t(n) = \sum_{j=0}^{N-t} \delta^j u(n+j) + \frac{\delta^{N-t+1}}{1-\delta} u(n+N-t).$$

Note that $V^N(n) = \frac{u(n)}{1-\delta}$ and, for $1 \leq t \leq N-1$,

$$V^t(n) = u(n) + \delta V^{t+1}(n+1). \tag{10}$$

We now want to show that the same is true in period T. Suppose n agents have already joined, with $0 \leq n \leq T-1$. When the platform charges P in period T, if agent T joins and the other n agents remain on the platform this period, the induction hypothesis implies that each participating agent's payoff is

$$b(n+1) - P + \delta V^{T+1}(n+1),$$

given that in all subsequent periods, every new agent is attracted and all other agents remain on the platform. This is an equilibrium iff agent T indeed wants to join and none of the n agents that have previously joined want to leave in period T. The payoff to such a deviation is $V^T(n)$ because by the induction hypothesis, all other agents remain on the platform in all subsequent periods and every new agent is attracted and stays. Thus, in period T all existing agents stay and the new agent joins iff

$$b(n+1) - P + \delta V^{T+1}(n+1) \geq V^T(n).$$

Using (10), this inequality is equivalent to

$$P \leq b(n+1) - u(n) = \Delta(n+1).$$

If the platform sets a higher price in period T, i.e., $P > \Delta(n+1)$, then in equilibrium no agents participate. To see this, suppose $k \in \{1, \ldots, n+1\}$ agents participate, so we must have

$$b(k) - P + \delta V^{T+1}(k) \geq u(k-1) + \delta V^{T+1}(k).$$

Indeed, if one of the k agents deviates to not participating this period, it obtains $u(k-1)$ this period and $V^{T+1}(k)$ from the perspective of next period by the induction hypothesis. Thus, we must have

$$b(k) - u(k-1) \geq P,$$

which implies

$$\Delta(n+1) \geq \Delta(k) \geq P,$$

a contradiction. Thus, when $P > \Delta(n+1)$, the only possible equilibrium in period T is that no agents participate. Thus, the platform's optimal price in period T is

$$P^T(n) = \Delta(n+1) > 0.$$

At this price, each participating agent's PDV payoff is

$$b(n+1) - \Delta(n+1) + \delta V^{T+1}(n+1) = V^T(n).$$

Since this is profitable for the platform in every period, induction logic establishes that in every period $t \in \{1, \ldots, N\}$, if $n \in \{0, \ldots, t-1\}$ agents have already joined, the platform can induce the selected agent to join and all existing agents to remain by setting $P^t(n) = \Delta(n+1)$.

On the equilibrium path the number of existing agents is $n = t-1$ at the beginning of each period $t \in \{1, \ldots, N\}$, and $n = N$ for all $t \geq N+1$. On the equilibrium path, the platform charges $P^t = \Delta(t)$ for $1 \leq t \leq N$ and $P^t = \Delta(N)$ for $t \geq N+1$. All agents join the platform and remain on it along the equilibrium path.

The PDV of payoffs for agent k along the equilibrium path is

$$\begin{aligned} W(k) &= \sum_{t=1}^{k-1} \delta^{t-1} u(t) + \delta^{k-1} V^{k}(k-1) \\ &= \sum_{t=1}^{k-1} \delta^{t-1} u(t) + \sum_{t=k}^{N} \delta^{t-1} u(t-1) + \frac{\delta^{N}}{1-\delta} u(N-1). \end{aligned}$$

Since $u(n)$ is decreasing (strictly decreasing for at least one n within $0 \leq n \leq N-1$), $W(k)$ is strictly lower than each agent's payoff in the absence of the platform, which is simply $\frac{u(0)}{1-\delta}$. Moreover, it is easily verified that $W(k)$ is weakly decreasing in k, so agents that are selected earlier obtain weakly higher PDV payoffs, strictly so whenever $u(k-1) > u(k)$ for some k. Specifically, in this case, the difference in PDV payoff between agent k and agent $k+1$ is $\delta^{k-1}(u(k-1) - u(k)) > 0$. This is because an agent's value of not joining worsens as other agents join, so agents selected later are made indifferent against a weaker outside option.

Finally, note that if we interpret agents' payoffs as $\delta \to 1$, then the term that dominates payoffs is the eventual payoff (after the initial $N-1$ periods) which is $u(N-1)$ in all subsequent periods, coinciding exactly with the payoffs in our one-period model in the case without pivotal agents.

A.2 All pivotal agents

Next we suppose the first $N-1$ agents are pivotal by assuming $\Delta(N-1) < 0 < \Delta(N)$. In period $t = N+1$, if $n < N$ have previously joined, then the maximum price that the platform can charge is $P = \Delta(n) < 0$, which is not profitable, so the platform attracts no agents from here on out, and $\Pi^{N+1}(n) = 0$ and $V^{N+1}(n) = \frac{u(0)}{1-\delta}$. If $n = N$ agents have joined, then it is easily seen the platform will charge $P = \Delta(N)$ and attract all agents, leading to $\Pi^{N+1}(N) = \frac{N\Delta(N)}{1-\delta}$ and $V^{N+1}(N) = \frac{u(N-1)}{1-\delta}$.

Now consider period $t = N$. If $n < N-1$ agents have been previously attracted, then the platform will attract no agents from next period onwards and the maximum price it can hope to charge this period is $P = \Delta(n+1) < 0$. Thus, the platform attracts no one in this period and in any of the subsequent periods, leading to $\Pi^{N}(n) = 0$ and $V^{N}(n) = \frac{u(0)}{1-\delta}$.

If $n = N-1$ agents have been previously attracted, then the platform attracts all N agents iff

$$b(N) - P + \frac{\delta u(N-1)}{1-\delta} \geq u(N-1) + \frac{\delta u(0)}{1-\delta}.$$

Indeed, if one agent deviates to not participating this period, then next period and thereafter the platform will attract no agents. Thus, the platform attracts all N agents in period N iff

$$P \leq P^{N} = \Delta(N) - \frac{\delta}{1-\delta}\left(u(0) - u(N-1)\right).$$

This is profitable iff

$$NP^{N} + \delta \Pi^{N+1}(N) > 0,$$

which is equivalent to

$$\Delta(N) > \delta\left(u(0) - u(N-1)\right).$$

Furthermore, it cannot be profitable for the platform to set a higher price, i.e., $P > P^{N}$. If it were, then suppose the platform attracts $k \in \{1, \ldots, N-1\}$ agents to participate with $P > P^{N}$ (we already know it cannot attract N agents). Then we must have

$$b(k) - P + \frac{\delta u(0)}{1-\delta} \geq u(k-1) + \frac{\delta u(0)}{1-\delta},$$

which implies $P \leq \Delta(k) < 0$. This cannot be profitable because in this case the platform attracts no agents in any subsequent period (since $k < N$).

Thus, if $n = N-1$ agents have been previously attracted and

$$\Delta(N) > \delta\left(u(0) - u(N-1)\right),$$

then the platform optimally attracts all agents in period N and every period thereafter, leading to the following PDV of profits and agent surplus in period N

$$\begin{aligned} \Pi^{N}(N-1) &= \frac{N\left(\Delta(N) - \delta\left(u(0) - u(N-1)\right)\right)}{1-\delta} \\ V^{N}(N-1) &= u(N-1) + \frac{\delta u(0)}{1-\delta}. \end{aligned}$$

Otherwise, the platform does not attract any agents in period N and all periods thereafter, so

$$\begin{aligned} \Pi^{N}(N-1) &= 0 \\ V^{N}(N-1) &= \frac{u(0)}{1-\delta}. \end{aligned}$$

For all $t \in \{1, \ldots, N-1\}$, define

$$\pi(t) \equiv \sum_{k=0}^{N-1-t} \delta^{k}(k+t)\left(\Delta(k+t) - \delta\left(u(0) - u(k+t)\right)\right) + \delta^{N-t}\frac{N\left(\Delta(N) - \delta\left(u(0) - u(N-1)\right)\right)}{1-\delta}.$$

Suppose the following induction hypothesis holds for period $T+1 \leq N-1$. If at the start of period $T+1$ the platform has $n \leq T$ existing agents, then

$$\begin{aligned}
\Pi^{T+1}(n) &= \begin{cases} \pi(T+1) & \text{if } \quad n = T \text{ and } \pi(T+1) > 0 \\ 0 & \text{if } \quad n < T \text{ or } \pi(T+1) \leq 0 \end{cases} \\
V^{T+1}(T) &= \begin{cases} u(T) + \frac{\delta u(0)}{1-\delta} & \text{if } \quad n = T \text{ and } \pi(T+1) > 0 \\ \frac{u(0)}{1-\delta} & \text{if } \quad n < T \text{ or } \pi(T+1) \leq 0 \end{cases},
\end{aligned}$$

We now want to show that the same holds true in period T. Suppose n agents have already joined at the start of period T, with $0 \leq n \leq T-1$. If $n < T-1$, then we know from the induction hypothesis that the platform makes zero profits from next period onwards and in the current period the maximum price it can charge is $P = \Delta(n+1) < 0$, so the platform attracts no agents this period either and therefore

$$\begin{aligned}
\Pi^T(n) &= 0 \\
V^T(n) &= \frac{u(0)}{1-\delta}.
\end{aligned}$$

Now suppose $n = T-1$. The platform attracts all T agents in this period iff

$$b(T) - P + \delta V^{T+1}(T) \geq u(T-1) + \frac{\delta u(0)}{1-\delta}.$$

If $\pi(T+1) \leq 0$, then the platform makes zero profits from next period onwards, and we must have $P \leq \Delta(T) \leq \Delta(N-1) < 0$, so this cannot be profitable. Thus, attracting all T agents this period can only be profitable if $\pi(T+1) > 0$, so $V^{T+1}(T) = u(T) + \frac{\delta u(0)}{1-\delta}$. Then the platform attracts all T agents in period T iff

$$P \leq P^T = \Delta(T) - \delta(u(0) - u(T)).$$

Furthermore, it cannot be profitable for the platform to set a higher price, i.e., $P > P^T$. If it were, then suppose the platform attracts $k \in \{1, \dots, T-1\}$ agents to participate with $P > P^T$ (we already know it cannot attract all T agents). Then we must have

$$b(k) - P + \frac{\delta u(0)}{1-\delta} \geq u(k-1) + \frac{\delta u(0)}{1-\delta},$$

which implies $P \leq \Delta(k) < 0$. This cannot be profitable because in this case the platform attracts no agents in any subsequent period (since $k < T < N$).

Thus, the platform optimally sets $P = P^T$ in period T and attracts all agents. This is profitable iff

$$TP^{T}+\delta\Pi^{T+1}\left(T\right)>0.$$

From the induction hypothesis, we have $\Pi^{T+1}\left(T\right)=\pi\left(T+1\right)$, so

$$\begin{aligned}
TP^{T}+\delta\Pi^{T+1}\left(T\right) &= T\left(\Delta\left(T\right)-\delta\left(u\left(0\right)-u\left(T\right)\right)\right) \\
&\quad +\sum_{k=0}^{N-2-T}\delta^{k+1}\left(k+T+1\right)\left(\Delta\left(k+T+1\right)-\delta\left(u\left(0\right)-u\left(k+T+1\right)\right)\right) \\
&\quad +\delta^{N-T}\frac{N\left(\Delta\left(N\right)-\delta\left(u\left(0\right)-u\left(N-1\right)\right)\right)}{1-\delta}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{k=0}^{N-1-T}\delta^{k}\left(k+T\right)\left(\Delta\left(k+T\right)-\delta\left(u\left(0\right)-u\left(k+T\right)\right)\right)+\delta^{N-T}\frac{N\left(\Delta\left(N\right)-\delta\left(u\left(0\right)-u\left(N-1\right)\right)\right)}{1-\delta} \\
&= \pi\left(T\right).
\end{aligned}$$

Thus, attracting all T agents in period T with $P=P^{T}$ is profitable iff $\pi\left(T\right)>0$, otherwise the platform attracts no agents from period T onwards and makes zero profits. Note also that $P^{T}<0$, so $\pi\left(T\right)>0$ implies $\pi\left(T+1\right)>0$. And if $\pi\left(T\right)>0$, then

$$V^{T}\left(T-1\right)=u\left(T-1\right)+\frac{\delta u\left(0\right)}{1-\delta}.$$

Otherwise, $\Pi^{T}\left(T-1\right)=0$ and $V^{T}\left(T-1\right)=\frac{u(0)}{1-\delta}$.

By induction, this is true for all $T\in\left\{1,\ldots,N-1\right\}$. Thus, if $\pi\left(1\right)\leq0$, then the platform cannot profitably attract any agents in any period, so it makes zero profits and each agent's PDV of payoffs is $\frac{u(0)}{1-\delta}$. If on the other hand $\pi\left(1\right)>0$, then along the equilibrium path the platform charges $P^{t}=\Delta\left(t\right)-\delta\left(u\left(0\right)-u\left(t\right)\right)$ for $1\leq t\leq N$ and $P^{t}=\Delta\left(N\right)$ for $t\geq N+1$. All agents join the platform and remain on it along the equilibrium path. In this case the platform's PDV of profits is

$$\Pi=\pi\left(1\right)=\sum_{k=1}^{N-1}\delta^{k-1}k\left(\Delta\left(k\right)-\delta\left(u\left(0\right)-u\left(k\right)\right)\right)+\delta^{N-1}\frac{N\left(\Delta\left(N\right)-\delta\left(u\left(0\right)-u\left(N-1\right)\right)\right)}{1-\delta}>0$$

and the PDV of payoffs for agent n along the equilibrium path is

$$W\left(n\right)=\sum_{k=1}^{n-1}\delta^{k-1}u\left(k\right)+\delta^{n-1}V^{n}\left(n-1\right)=\sum_{k=1}^{n-1}\delta^{k-1}u\left(k\right)+\delta^{n-1}u\left(n-1\right)+\frac{\delta^{n}u\left(0\right)}{1-\delta}.$$

Since $u(k)$ is decreasing (strictly decreasing for at least one k within $0 \leq k \leq N-1$), $W(n)$ is lower than each agent's payoff in the absence of the platform, strictly lower at least for agent N. Moreover, $W(n)$ is weakly decreasing in n, so agents that are selected earlier obtain weakly higher PDV payoffs.

Note that as $\delta \to 1$, the condition for the platform to be able to profitably attract all agents ($\pi(1) > 0$) becomes simply $b(N) > u(0)$. This assumes, as we assumed throughout the paper, that the platform does not face any financial constraints, so it could set a negative price for any finite number of periods provided the PDV of prices is positive. If instead there were a financial constraint on the platform that it must turn positive profits within a certain number of periods, this would clearly reduce the parameter range in which the platform can attract all agents and thereby engineer a platform trap.

B Commitment to prices

Let's consider the general setting of Section 5, of which the marketplace baseline setting is a special case. We first show that committing to a fixed sequence of prices independent of agents' participation decisions is often worse for the platform than retaining the flexibility to adjust its offers. An exception arises if agents are pivotal and $b(\cdot)$ is decreasing. As shown below, committing to a sequence of decreasing prices can render earlier agents non-pivotal and increase the platform's profits and the scope for a platform trap. We then show full commitment to participation-contingent pricing — if possible — always maximizes the platform's profit.

Suppose the platform commits upfront to the N prices that will be charged to the N agents, who still arrive sequentially.

Proposition 9. *If $b(\cdot)$ is weakly increasing, then the platform's profit-maximizing prices (with commitment) are $P^k = b(N) - u(k-1)$ for $k = 1, \ldots, N$, and it profitably attracts all agents iff*

$$Nb(N) - \sum_{k=0}^{N-1} u(k) > 0.$$

If $b(\cdot)$ is single-peaked such that $\max_{k \leq N} \{b(k)\} = b(n_0)$, with $1 \leq n_0 \leq N$, then the platform's profit-maximizing prices are

$$P^k = \begin{cases} b(N+1-k) - u(N-k) & \text{if} \quad 1 \leq k \leq N - n_0 \\ b(n_0) - u(k - (N - n_0 + 1)) & \text{if} \quad N - n_0 + 1 \leq k \leq N \end{cases},$$

and it profitably attracts all agents iff

$$\sum_{k=0}^{N-n_0-1} b\left(N-k\right)+n_0 b\left(n_0\right)-\sum_{k=0}^{N-1} u\left(k\right)>0.$$

Proof: We start by proving Lemma 2 for the case when the platform commits to its prices, namely that it is optimal to either attract all agents or none of them. Suppose to the contrary, the platform finds it optimal to only attract $0<n<N$ agents. In this case, the highest price that the platform can charge to any participating agent is $\Delta\left(n\right)$ and so we must have $\Delta\left(n\right)>0$, otherwise the platform would not find it optimal to attract n agents. Now consider any agent that does not join at its optimal prices (it must exist) and suppose it is the k-th agent. If the platform keeps all prices unchanged except the price to agent k, which is changed to $P^k=\Delta\left(n+1\right)\geq\Delta\left(n\right)>0$, then agent k will participate and all agents that were joining previously, when agent k did not join, will continue to join. So the platform has strictly increased profits, which means it couldn't have been optimal to only induce $n<N$ agents to participate.

We now proceed by induction for $N\geq 2$. Start with $N=2$. If the platform finds it optimal to attract any agents, it must attract both. And there are two possibilities for the platform's profit-maximizing prices that attract both agents. If the first agent is pivotal given the platform's prices, then we must have $P^1=b\left(2\right)-u\left(0\right)$. This implies the profit-maximizing price for the second agent is $P^2=b\left(2\right)-u\left(1\right)$. If on the other hand the first agent is not pivotal given the platform's prices, then we must have $P^1=b\left(2\right)-u\left(1\right)$. In this case, the second agent must join even if the first agent does not join, so the profit-maximizing price for the second agent is $P^2=b\left(1\right)-u\left(0\right)$. Thus, if $b\left(2\right)\geq b\left(1\right)$, then the platform's profit-maximizing prices are $P^1=b\left(2\right)-u\left(0\right)$ and $P^2=b\left(2\right)-u\left(1\right)$, resulting in total profits $2b\left(2\right)-u\left(0\right)-u\left(1\right)$. If on the other hand $b\left(2\right)<b\left(1\right)$, then the platform's profit-maximizing prices are $P^1=b\left(2\right)-u\left(1\right)$ and $P^2=b\left(1\right)-u\left(0\right)$, resulting in total profits $b\left(1\right)+b\left(2\right)-u\left(0\right)-u\left(1\right)$. So the result holds for $N=2$.

Now suppose the result holds for $N=n\geq 2$, and suppose $N=n+1$ and $b\left(\cdot\right)$ is single-peaked such that $\max_{1\leq k\leq n+1}\left\{b\left(k\right)\right\}=b\left(n_0\right)$, with $1\leq n_0\leq n+1$. There are two possibilities for the platform's profit-maximizing prices that attract all agents. If the first agent is pivotal given these prices, then we must have $P^1=b\left(n+1\right)-u\left(0\right)$. This ensures the first agent joins, so we can apply the induction hypothesis to the remaining n agents and their prices $P^2,\ldots,P^{n+1}$, with payoff functions $\widetilde{b}\left(k\right)=b\left(k+1\right)$ and $\widetilde{u}\left(k\right)=u\left(k+1\right)$, so that $\widetilde{b}\left(\cdot\right)$ is single-peaked and

$$\max_{1\leq k\leq n}\left\{\widetilde{b}\left(k\right)\right\}=\max_{2\leq k\leq n+1}\left\{b\left(k\right)\right\}=b\left(\max\left\{n_0,2\right\}\right)=\widetilde{b}\left(\max\left\{n_0,2\right\}-1\right),$$

where we have used that if $n_0=1$, then $b\left(n\right)$ must be decreasing, and so $\max_{2\leq k\leq n+1}\left\{b\left(k\right)\right\}=b\left(2\right)$.

We therefore must have

$$\begin{aligned}
P^k &= \begin{cases} \widetilde{b}\left(n+2-k\right)-\widetilde{u}\left(n+1-k\right) & \text{if} \quad 2\leq k\leq n+2-\max\left\{n_0,2\right\} \\ \widetilde{b}\left(\max\left\{n_0,2\right\}-1\right)-\widetilde{u}\left(k-\left(n+3-\max\left\{n_0,2\right\}\right)\right) & \text{if} \quad n+3-\max\left\{n_0,2\right\}\leq k\leq n+1 \end{cases} \\
&= \begin{cases} b\left(n+3-k\right)-u\left(n+2-k\right) & \text{if} \quad 2\leq k\leq n+2-\max\left\{n_0,2\right\} \\ b\left(\max\left\{n_0,2\right\}\right)-u\left(k-\left(n+2-\max\left\{n_0,2\right\}\right)\right) & \text{if} \quad n+3-\max\left\{n_0,2\right\}\leq k\leq n+1 \end{cases} \qquad (11)
\end{aligned}$$

So in this case total profits are

$$\begin{aligned}
& b\left(n+1\right)+\sum_{k=2}^{n+2-\max\{n_0,2\}} b\left(n+3-k\right)+\left(\max\left\{n_0,2\right\}-1\right)b\left(\max\left\{n_0,2\right\}\right)-\sum_{k=0}^{n}u\left(k\right) \\
= \; & b\left(n+1\right)+\sum_{k=\max\{n_0,2\}+1}^{n+1} b\left(k\right)+\left(\max\left\{n_0,2\right\}-1\right)b\left(\max\left\{n_0,2\right\}\right)-\sum_{k=0}^{n}u\left(k\right). \qquad (12)
\end{aligned}$$

Now suppose the first agent is not pivotal given the platform's profit-maximizing prices. In this case, we must have $P^1=b\left(n+1\right)-u\left(n\right)$. And the remaining n agents must join given the platform's prices even if the first agent does not join. Thus, applying the induction hypothesis to agents $k=2,\ldots,n+1$ and noting that $\max_{1\leq k\leq n}\left\{b\left(k\right)\right\}=b\left(\min\left\{n_0,n\right\}\right)$ (recall $b\left(\cdot\right)$ is single-peaked), we must have

$$P^k=\begin{cases} b\left(n+2-k\right)-u\left(n+1-k\right) & \text{if} \quad 2\leq k\leq n+1-\min\left\{n_0,n\right\} \\ b\left(\min\left\{n_0,n\right\}\right)-u\left(k-\left(n-\min\left\{n_0,n\right\}+2\right)\right) & \text{if} \quad n-\min\left\{n_0,n\right\}+2\leq k\leq n+1 \end{cases}.$$

So in this case total profits are

$$\begin{aligned}
& b\left(n+1\right)+\sum_{k=2}^{n+1-\min\{n_0,n\}} b\left(n+2-k\right)+\min\left\{n_0,n\right\}b\left(\min\left\{n_0,n\right\}\right)-\sum_{k=0}^{n}u\left(k\right) \\
= \; & \sum_{k=\min\{n_0,n\}+1}^{n+1} b\left(k\right)+\min\left\{n_0,n\right\}b\left(\min\left\{n_0,n\right\}\right)-\sum_{k=0}^{n}u\left(k\right).
\end{aligned}$$

Suppose $b\left(\cdot\right)$ is weakly increasing, which means $n_0=n+1$. If the first agent is pivotal at the platform's profit-maximizing prices, we have from the first case above

$$P^k=b\left(n+1\right)-u\left(k-1\right)$$

for $1 \leq k \leq n+1$ and total profits

$$(n+1)\, b\,(n+1) - \sum_{k=0}^{n} u\,(k)\,.$$

If the first agent is not pivotal at the platform's profit-maximizing prices, we have from the second case above

$$\begin{aligned} P^1 &= b\,(n+1) - u\,(n) \\ P^k &= b\,(n) - u\,(k-2) \;\text{ for } 2 \leq k \leq n+1 \end{aligned}$$

and total profits

$$b\,(n+1) + nb\,(n) - \sum_{k=0}^{n} u\,(k)\,.$$

Since $b\,(\cdot)$ is increasing, we have

$$b\,(n+1) + nb\,(n) - \sum_{k=0}^{n} u\,(k) \leq (n+1)\, b\,(n+1) - \sum_{k=0}^{n} u\,(k)\,,$$

which implies the profit-maximizing prices must be $P^k = b\,(n+1) - u\,(k-1)$ for $1 \leq k \leq n+1$, so every agent (including the first one) is pivotal.

Now suppose $b\,(\cdot)$ is single-peaked and $\max_{1\leq k\leq n+1}\{b\,(k)\} = b\,(n_0)$, with $1 \leq n_0 < n+1$. In this case, if the first agent is pivotal at the platform's profit-maximizing prices, then profits are given by expression (12) above. If the first agent is not pivotal at the platform's profit-maximizing prices, then we have from the second case above

$$P_k = \begin{cases} b\,(n+2-k) - u\,(n+1-k) & \text{if} \quad 1 \leq k \leq n+1-n_0 \\ b\,(n_0) - u\,(k-(n-n_0+2)) & \text{if} \quad n-n_0+2 \leq k \leq n+1 \end{cases}\,. \tag{13}$$

and total profits are

$$\sum_{k=n_0+1}^{n+1} b\,(k) + n_0 b\,(n_0) - \sum_{k=0}^{n} u\,(k)\,.$$

Comparing this expression with (12), we have

$$\begin{aligned} & \sum_{k=n_0+1}^{n+1} b(k) + n_0 b(n_0) - \sum_{k=0}^{n} u(k) \qquad (14) \\ \geq \; & b(n+1) + \sum_{k=\max\{n_0,2\}+1}^{n+1} b(k) + (\max\{n_0,2\}-1)\, b(\max\{n_0,2\}) - \sum_{k=0}^{n} u(k) \end{aligned}$$

for any n_0 such that $1 \leq n_0 \leq n$. Indeed, if $n_0 = 1$, so $b(\cdot)$ is decreasing, then the inequality (14) reduces to

$$b(1) \geq b(n+1),$$

which is clearly true. If $2 \leq n_0 \leq n$, then the inequality (14) reduces to

$$b(n_0) \geq b(n+1),$$

which is also true. Thus, when $b(\cdot)$ is single-peaked such that $\max_{1\leq k \leq n+1}\{b(k)\} = b(n_0)$, with $n_0 \leq n$, the profit-maximizing platform prices are given by (13) above.

Thus, the induction hypothesis holds for $N = n+1$ when $b(\cdot)$ is weakly increasing or single-peaked, so it holds for all $N \geq 2$. ■

A few observations follow from this proposition.

- When $b(\cdot)$ is weakly increasing, all agents are pivotal when the platform commits to prices (Proposition 9), so the platform's profits are weakly lower than in the case without commitment (Proposition 1 or Proposition 3 in the baseline), strictly so if not all agents are pivotal in the baseline.
- When $b(\cdot)$ is decreasing, we have

  $$P^k = b(n+2-k) - u(n+1-k)$$

  for $1 \leq k \leq n+1$, so prices are weakly decreasing in k because $\Delta(\cdot)$ is weakly increasing. So with commitment, the first agent gets charged the highest price. Moreover, profit can now be higher than in the baseline case without commitment, depending on how many agents are pivotal in the baseline. E.g. profit is now strictly higher in the case that all agents are pivotal in the baseline. The logic behind this is that the platform commits to attract later agents (with a lower price) even if earlier agents don't join, so it takes away the pivotal role of earlier agents. The result is prices are decreasing rather than increasing. This strategy is profitable when attracting later agents in case earlier agents don't join is not too costly, which arises when $b(\cdot)$ is decreasing, indeed meaning the platform can extract more in total by committing to these prices.

Now, suppose instead, the platform can commit to an optimal contingent price schedule. Then the platform can always achieve maximal payoffs even without agent-specific pricing. As shown in the proposition below, it can do so by using second-degree price discrimination; i.e., by committing to a contingent price function $P(j)$, where all participating agents pay the same price based on the total number j of agents that join.[3] However, this approach may require the platform to commit to selling at a loss off the equilibrium path; i.e., if only $n < N$ agents join and $\Delta(n) < 0$. Moreover, even if $\Delta(1) > 0$, implementing contingent pricing mechanisms is likely to be difficult in practice.

Proposition 10. *If the platform can commit to a single contingent pricing function offered to all agents, it maximizes profits by offering each agent the same contingent price* $P(j) = \Delta(j)$ *where* $j \in \{1, \ldots, N\}$ *is the number of agents that end up joining. This induces all* N *agents to join and yields a profit of* $N\Delta(N)$*. All agents would be strictly better off without the platform.*

Proof: Suppose the platform offers the pricing function $P(j) = \Delta(j)$ for all $j = 1, \ldots, N$. The last agent will pay $\Delta(N' + 1)$ if they join and N' prior agents have joined, so they join (they are indifferent between joining and not). Knowing the last agent will join no matter what, the second-to-last agent will pay $\Delta(N' + 2)$ if they join and N' prior agents have joined, which yields the same payoff $u(N' + 1)$ as not joining. So the second-to-last agent joins too. Thus, by backwards induction, all agents join and along the equilibrium path they are all charged $\Delta(N)$. This is the maximum price that the platform can charge any agent while getting all agents to join. ■

C Ad valorem fees

We modify the model so there is positive pass-through from the fee a platform charges a seller to the seller's price. This provides an additional mechanism by which buyers could ultimately end up being worse off because of the platform. To do so we assume the platform charges ad valorem fees (a percentage of a seller's revenue) instead of a fixed fee per seller. When deciding whether to search on the platform or in the direct channel, we continue to assume buyers observe how many sellers have listed on the platform, but they do not observe the fees the sellers are charged or the seller prices.

Let the ad valorem fee charged to a seller be denoted τ, where $0 \leq \tau \leq 1$. Facing such a fee, a seller's per-buyer profit from sales on the platform is $\max_p \{((1-\tau)p - c)\, q_P(p)\}$. Let $p(\tau) =$

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[3] Similarly, the platform could replicate the maximum platform trap from Proposition 10 if it can commit to run an auction in each stage for all unsigned agents, inducing them to compete for joining earlier. We showed this formally in an earlier version of the paper.

$\arg\max_p \{((1-\tau)p-c)\,q_P(p)\}$, which is increasing in τ. Then $\pi_P(\tau) = ((1-\tau)p(\tau)-c)\,q_P(p(\tau))$ is a seller's profit per buyer, which is decreasing in τ, and $b(n,\tau) \equiv \frac{1}{N} m(n)\,\pi_P(\tau)$ is a seller's expected profit from sales on the platform when n sellers join the platform. Meanwhile, a seller's expected sales from the direct channel is $u(n) = \frac{1}{N}(1-m(n))\,\pi_D$, where π_D is a seller's profit per buyer in the direct channel.

Given a buyer's indirect utility function $v_P(p)$, a buyer's indirect utility from purchasing from her matched seller on the platform is $v_P(\tau) = v_P(p(\tau))$, which is also decreasing in τ. For prices where either quantity is positive, we assume $q_P(p) > q_D(p)$, so the platform is more efficient at making transactions, which implies $\pi_P(0) > \pi_D$ and $v_P(0) > v_D$. Let the buyers' draws of costs to go to each channel be such that $s_P = 0$ and $s = s_D$ is distributed according to $G(\cdot)$ with full support over $[0, \overline{s}]$, so buyers face costs of going to the direct channel but not to going to the platform. We assume $v_D > \overline{s}$.

Suppose buyers expect a single equilibrium fee τ^*. If n sellers participate on the platform, a buyer will go to the platform if their expected utility $\frac{n}{N} v_P(\tau^*)$ is greater than the expected utility of going to the direct channel, which is $\frac{N-n}{N} v_D - s$. Note since a buyer's expected utility from going to the platform is non-negative, they will never choose the outside option. Thus, the measure of buyers going to the platform is

$$m(n) = 1 - G\left(\frac{N-n}{N} v_D - \frac{n}{N} v_P(\tau^*)\right), \tag{15}$$

with $1 - m(n)$ going to the direct channel, where this is bounded between zero and one. In this setting, $m(n)$ is weakly increasing in n, with $m(0) = 0$ and $m(N) = 1$. We assume τ^* is such that

$$\frac{1}{N} v_D - \frac{N-1}{N} v_P(\tau^*) < \overline{s}$$

so that $m(N-1) > 0$ and therefore $u(N-1) < u(0)$ holds.

Furthermore, to simplify the analysis, suppose when $\tau = 0$, we have $m(1)\,\pi_P(0) > \pi_D$, which means $b(1,0) > u(0)$: this rules out sellers being pivotal when the platform charges no fee (because a seller obtains higher expected profit on the platform than the direct channel if it was charged no fee even if it is the only seller to join the platform).

Consider the last seller to decide, facing the fee τ^N and knowing N' other sellers have already joined the platform. If the seller joins, it will get

$$\frac{m(N'+1)\,\pi_P\left(\tau^N\right)}{N}$$

while if it does not join, it will get

$$\frac{(1-m(N'))\,\pi_D}{N}.$$

Since $m(1)\,\pi_P(0) > \pi_D$ and $m(\cdot)$ is weakly increasing, we have $m(N'+1)\,\pi_P(0) > (1-m(N'))\,\pi_D$ and $m(N'+1)\,\pi_P(1) = 0 \leq (1-m(N'))\,\pi_D$. Recalling that $\pi_P(\tau)$ is decreasing, this implies the platform will profitably attract the last seller with a fee $\tau^N(N') > 0$, which is uniquely defined by

$$\pi_P(\tau) = \frac{1-m(N')}{m(N'+1)}\pi_D.$$

A very similar reasoning ensures that the platform can profitably induce the second-to-last seller to participate no matter how many previous sellers have joined, and so on, all the way back to the first seller. Thus, by the same logic as in our baseline analysis, the platform can induce all sellers to join by charging appropriate fees to each seller. The equilibrium fee is $\tau^* = \tau^N(N-1) > 0$, given each seller expects all other sellers to join in equilibrium.

In the equilibrium with the platform, sellers end up with expected payoffs $\frac{1}{N}\pi_P(\tau^*)$, which after using the equilibrium characterization of τ^* equals $\frac{1}{N}(1-m(N-1))\,\pi_D$. This compares to getting $\frac{1}{N}\pi_D$ in the absence of the platform. So clearly all sellers are strictly worse off given $m(N-1) > 0$.

Now consider buyers' expected utility. If the platform didn't exist, they would get $v_D - s$, with s drawn from distribution G on $[0, \overline{s}]$. With the platform they get $v_P(\tau^*)$. Thus, some buyers are strictly worse off with the platform provided $v_P(\tau^*) < v_D$, while all buyers are strictly worse off provided $v_P(\tau^*) < v_D - \overline{s}$. E.g., this last inequality together with the conditions above all hold (so that all buyers and sellers are strictly worse off due to the existence of the platform) if we assume each buyer chooses q on channel $j \in \{P, D\}$ to maximize the quasi-linear net utility form $\alpha_j q - \frac{1}{2}q^2 - pq$ (so that $q_j(p) = \alpha_j - p$), and $N = 10$, $c = 4$, $\alpha_D = 6$, $\alpha_P = 8$ and $\overline{s} = \frac{2}{5}$ (note, in this case $\tau^* \simeq 0.4632$).

D Competing sellers

Our model with independent sellers can be extended to capture the possibility of some competition between sellers. This allows the possibility that $b(n)$ may be decreasing in n. To do so, we modify the previous setting so rather than having elastic demand for a matching seller's product, buyers only want to buy one unit of a product, which they value at v, i.e., they get this same value from any of the N sellers. Buyers are initially informed of a single seller, randomly drawn from the N sellers. When a buyer goes to channel $j \in \{P, D\}$, if there are $k \geq 1$ sellers there, with probability $f_j(k)$ a buyer becomes informed of all other sellers on the channel regardless of whether the seller the buyer is initially informed of is among these sellers,[4] and buys from the seller on the channel with the lowest price among the ones that the buyer is informed of (randomizing if there is more than one such seller). We assume f_j is an increasing function of k that is bounded between zero and one over $1 \leq k \leq N$. With probability $1 - f_j(k)$ the buyer does not discover the other sellers on the channel, and therefore can only buy from the seller it is initially informed of if that seller is on the channel. This is a simple way to introduce increased price competition as the number of sellers participating on a channel increases. We retain all other assumptions as in the baseline model: buyers do not know which sellers are on which channel (or their prices) until they go to a channel, but they do know how many sellers are on each channel; buyers have an outside option (in case they do not buy from any seller) normalized to zero; and each seller has marginal cost c.

If $k = 1$ so there is only one seller on a channel, the seller never faces any competition and so it always prices at v, meaning buyers will get zero surplus on the channel. We assume $f_j(1) = 0$ on both channels, so if there is only one seller on a channel, the probability buyers are informed of the seller if it is not the seller they are initially informed about is normalized to zero. Thus, such a seller will obtain a profit of $\frac{1}{N}(v-c)$ per buyer on that channel, where here $\frac{1}{N}$ represents the probability that any given buyer is initially informed of the seller.

If there are $k \geq 2$ sellers at a channel, then sellers will price according to a standard mixed strategy equilibrium, with a seller's profit per buyer being $(1 - f_j(k))\frac{1}{N}(v-c)$, which is a seller's certain profit per buyer if it prices at v and only sells to a buyer who doesn't discover the other sellers (here we multiply by $\frac{1}{N}$, the probability the buyer is initially informed about it). This implies if n sellers join the platform, a seller's expected profit on the platform is $b(n) = (1 - f_P(n))\frac{1}{N}(v-c)\,m(n)$ and on the direct channel is $u(n) = (1 - f_D(N-n))\frac{1}{N}(v-c)(1-m(n))$, where $m(n)$ is the measure of buyers choosing the platform channel.

To work out $m(n)$, we need to work out a buyer's expected net utility on each channel (initially, ignoring the costs of going to the channel) when n sellers participate on the platform. Obviously, if a buyer goes to a channel where there are no sellers, then it will get zero expected net utility. The same is true if there is just one seller on the channel, since it prices at v. If, instead, there are $k \geq 2$ sellers on channel j, a buyer's expected net utility on channel j can be obtained by subtracting expected seller industry profit per buyer from expected total welfare per buyer. To calculate the latter, note that with probability $\left(1 - \frac{k}{N}\right)(1 - f_j(k))$ a consumer that comes to channel j ends up not transacting because the seller they know of initially is on the other channel and the consumer

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[4] This means if the seller they are initially informed of is not on the channel, the buyer becomes informed of k other sellers; while if it is on the channel, they become informed of $k-1$ other sellers.

does not discover other sellers on channel j. Thus, expected total welfare per buyer in channel j is

$$\left(1-\left(1-\frac{k}{N}\right)\left(1-f_j\left(k\right)\right)\right)\left(v-c\right),$$

and therefore expected net consumer surplus on channel j is

$$\begin{aligned} & \left(1-\left(1-\frac{k}{N}\right)\left(1-f_j\left(k\right)\right)\right)\left(v-c\right)-k\left(1-f_j\left(k\right)\right)\frac{1}{N}\left(v-c\right) \\ = \; & f_j\left(k\right)\left(v-c\right). \end{aligned}$$

Thus, if n sellers participate on the platform with $2 \leq n \leq N-2$, a buyer will go to the platform if their expected utility $f_P\left(n\right)\left(v-c\right)-s_P$ is greater than the expected utility of going to the direct channel, which is $f_D\left(N-n\right)\left(v-c\right)-s_D$.[5] As a result, the measure of buyers going to the platform is

$$m\left(n\right)=1-G\left(\left(f_D\left(N-n\right)-f_P\left(n\right)\right)\left(v-c\right)\right),$$

for $2 \leq n \leq N-2$, which is clearly increasing in n.

We first need to specify $b\left(n\right)$ and $u\left(n\right)$ for the extreme cases where there is at most one seller on one of the channels. In such cases, we have

$$\begin{aligned} m\left(0\right) &= 1-G\left(f_D\left(N\right)\left(v-c\right)\right) \\ m\left(1\right) &= 1-G\left(f_D\left(N-1\right)\left(v-c\right)\right) \\ m\left(N-1\right) &= 1-G\left(-f_P\left(N-1\right)\left(v-c\right)\right) \\ m\left(N\right) &= 1-G\left(-f_P\left(N\right)\left(v-c\right)\right) \end{aligned}$$

Note $b\left(0\right)$ and $u\left(N\right)$ are undefined since a seller can't be on the platform when $n=0$, and can't be in the direct channel if $n=N$. Then we have $b\left(1\right)=\frac{1}{N}\left(v-c\right)m\left(1\right)$ and $u\left(N-1\right)=\frac{1}{N}\left(v-c\right)\left(1-m\left(N-1\right)\right)$.

Given the functions G, f_D and f_P, we can explore the properties of $b\left(n\right)$, $u\left(n\right)$, and $\Delta\left(n\right)$. To illustrate what is possible, we consider the simple case where all three functions are linear. Specifically, we assume s_D is uniformly distributed on $\left[-\frac{\overline{s}}{2},\frac{\overline{s}}{2}\right]$, and $s_P=-s_D$. Note this assumption is consistent with buyers having a standard Hotelling-type channel preference. This implies $s=s_D-s_P=2s_D$ is uniformly distributed over $\left[-\overline{s},\overline{s}\right]$, i.e., $G\left(s\right)=\max\left\{\min\left\{\frac{s+\overline{s}}{2\overline{s}},1\right\},0\right\}$. And we assume $f_j\left(k\right)=\theta_j\frac{k-1}{N}$, with $0<\theta_D\leq\theta_P<1$ to capture the idea that the platform enables greater discoverability of the sellers than the direct channel. With this formulation, buyers always

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[5] We continue to assume the draws s_D and s_P are such that buyers always prefer to go to one of the channels over the outside option. We will note this holds for the distribution of the draws used below.

prefer to go to one channel because ignoring the s_D and s_P terms, they get non-negative surplus from going to either channel, and given $s_P = -s_D$, one of s_D and s_P is always non-negative. Note $f_j(1) = 0$ as assumed above, with f_j strictly increasing in k.

We can illustrate that this model can lead to all the properties of the baseline model holding except that $b(n)$ is no longer always weakly increasing. We consider two numerical examples. First suppose:

$$N = 10, \qquad \theta_D = 0.1, \qquad \theta_P = 0.1, \qquad \overline{s} = 1, \qquad v - c = 2,$$

so the competition effect on the two channels is the same. Then, as Table 1 below shows, $m(n)$ is strictly increasing, $b(n)$ is strictly increasing, $u(n)$ is strictly decreasing and $\Delta(n)$ is strictly increasing, with $b(10) > \frac{1}{10}\sum_{k=1}^{10} u(k-1)$ so all the properties required for Proposition 1 to hold apply, and there is a platform trap.

Table 1: Numerical example with $N = 10$, $\theta_D = 0.1$, $\theta_P = 0.1$, $\overline{s} = 1$, and $v - c = 2$

n	$m(n)$	$b(n)$	$u(n)$	$\Delta(n)$
0	0.41000	—	0.10738	—
1	0.42000	0.08400	0.10672	-0.02338
2	0.44000	0.08712	0.10416	-0.01960
3	0.46000	0.09016	0.10152	-0.01400
4	0.48000	0.09312	0.09880	-0.00840
5	0.50000	0.09600	0.09600	-0.00280
6	0.52000	0.09880	0.09312	0.00280
7	0.54000	0.10152	0.09016	0.00840
8	0.56000	0.10416	0.08712	0.01400
9	0.58000	0.10672	0.08400	0.01960
10	0.59000	0.10738	—	0.02338

Now consider the following numerical example, where the competition effect on the platform is much stronger:

$$N = 10, \qquad \theta_D = 0.1, \qquad \theta_P = 0.5, \qquad \overline{s} = 1, \qquad v - c = 2.$$

Under these parameter values, the implied values of $m(n)$, $b(n)$, $u(n)$, and $\Delta(n)$ are given in Table 2.

In this second example, $b(n)$ is single-peaked, increasing up to $n = 7$, and decreasing for $n > 8$, with its peak at $b(7) = b(8)$. However, despite this, $b(N) > \frac{1}{10}\sum_{k=1}^{10} u(k-1)$ still holds, so that Proposition 1 again applies, meaning a platform trap again arises in this case.

Table 2: Numerical example with $N = 10$, $\theta_D = 0.1$, $\theta_P = 0.5$, $\overline{s} = 1$, and $v - c = 2$

n	$m(n)$	$b(n)$	$u(n)$	$\Delta(n)$
0	0.4100	—	0.10738	—
1	0.4200	0.0840	0.10672	-0.02338
2	0.4800	0.0912	0.09672	-0.01552
3	0.5400	0.0972	0.08648	0.00048
4	0.6000	0.1020	0.07600	0.01552
5	0.6600	0.1056	0.06528	0.02960
6	0.7200	0.1080	0.05432	0.04272
7	0.7800	0.1092	0.04312	0.05488
8	0.8400	0.1092	0.03168	0.06608
9	0.9000	0.1080	0.02000	0.07632
10	0.9500	0.1045	—	0.08450

E Full equilibrium characterization for special cases

In this section, we fully characterize the equilibrium of the game for several special cases:

- $N = 2$
- $N = 3$
- Any $N \geq 3$, $b(n) = b_0 + nb$ and $u(n) = u_0 - nu$, with $b_0 \geq 0$, $b > \frac{N-3}{2}u > 0$ and $u_0 > Nu$.
- $N = 4$, $b(n) = b_0 + nb$ and $u(n) = u_0 - nu$, with $b_0 \geq 0$ and $u_0 > Nu$.

Suppose first $N = 2$. If $\Delta(1) \leq 0$, then the first agent is pivotal, so the highest prices the platform can charge to attract both agents are $P^1 = b(2) - u(0)$ to the first agent and $P^2 = b(2) - u(1)$ to the second agent. This is profitable iff $2b(2) - u(0) - u(1) > 0$. If $\Delta(1) > 0$, then no agent is pivotal, so the platform optimally charges $P^1 = P^2 = \Delta(2) > 0$ to both agents and makes positive profits. To sum up, platform profits are

$$\begin{cases} 0 & \text{if} \quad \max\left\{b(2) - \frac{u(0)+u(1)}{2}, b(1) - u(0)\right\} \leq 0 \\ 2b(2) - u(1) - u(0) & \text{if} \quad b(1) - u(0) \leq 0 < b(2) - \frac{u(0)+u(1)}{2} \\ 2(b(2) - u(1)) & \text{if} \quad b(1) - u(0) > 0 \end{cases}.$$

Now suppose $N = 3$. If the first agent doesn't join, from above, the platform attracts the two remaining agents if

$$b(2) - \frac{u(0) + u(1)}{2} > 0$$

or

$$b(1) - u(0) > 0.$$

If the first agent does join, the platform attracts the two remaining agents if

$$b(3) - \frac{u(1) + u(2)}{2} > 0$$

or

$$b(2) - u(1) > 0.$$

It is easily seen that

$$b(2) - u(1) \geq \max\left\{b(1) - u(0), b(2) - \frac{u(0) + u(1)}{2}\right\}$$

because $\Delta(2) \geq \Delta(1)$ and $u(\cdot)$ is weakly decreasing. Thus, if the platform attracts the last two agents when the first agent does not join, it necessarily attracts the last two agents after the first agent joins. There are therefore three relevant cases:

1. $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} > 0$, in which case the platform attracts the last two agents regardless of whether the first agent has joined or not
2. $\max\left\{b(1) - u(0), b(2) - \frac{u(0)+u(1)}{2}\right\} \leq 0 < \max\left\{b(2) - u(1), b(3) - \frac{u(1)+u(2)}{2}\right\}$, in which case the platform attracts the last two agents iff the first agent has joined
3. $\max\left\{b(2) - u(1), b(3) - \frac{u(1)+u(2)}{2}\right\} \leq 0$, in which case the platform cannot profitably attract any agent.

In the first case, the platform optimally charges $P^1 = P^2 = P^3 = \Delta(3) > 0$ and makes profits $3\Delta(3) > 0$.

In the second case, the first agent is pivotal so the platform optimally charges $P^1 = b(3) - u(0)$ to attract the first agent. Its maximum profits from the last two agents are then (applying the case $N = 2$ and taking into account the first agent has joined)

$$\begin{cases} 2b(3) - u(2) - u(1) & \text{if} \quad b(2) - u(1) \leq 0 < b(3) - \frac{u(1)+u(2)}{2} \\ 2(b(3) - u(2)) & \text{if} \quad b(2) - u(1) > 0 \end{cases},$$

so total profits are

$$\begin{cases} 3b(3) - u(2) - u(1) - u(0) & \text{if} \quad b(2) - u(1) \leq 0 < b(3) - \frac{u(1)+u(2)}{2} \\ 3b(3) - 2u(2) - u(0) & \text{if} \quad b(2) - u(1) > 0 \end{cases}.$$

And in the third case, the platform makes zero profits.

Combining all cases above, we have four final cases:

- if $\max\left\{b(1)-u(0), b(2)-\frac{u(0)+u(1)}{2}\right\}>0$, the platform attracts all three agents, none are pivotal and the platform's profits are $3\Delta(3)$
- if $\max\left\{b(1)-u(0), b(2)-\frac{u(0)+u(1)}{2}\right\}\leq 0<b(2)-u(1)$ and $b(3)>\frac{2u(2)+u(0)}{3}$, the platform attracts all three agents, only the first agent is pivotal, and the platform's profits are $3b(3)-2u(2)-u(0)>0$
- if $b(2)-u(1)\leq 0<b(3)-\frac{u(2)+u(1)+u(0)}{3}$, the platform attracts all three agents, the first two agents are pivotal, and the platform's profits are $3b(3)-u(2)-u(1)-u(0)>0$.
- otherwise the platform attracts no agents.

Next, we show that for general $N\geq 2$, the equilibrium (including the number of pivotal agents) can be fully characterized for linear increasing $b(\cdot)$ and linear decreasing $u(\cdot)$, provided we add additional restrictions on their respective slopes in relation to N.

Proposition 11. *Suppose $b(n)=b_0+bn$ and $u(n)=u_0-un$, where $b_0\geq 0$, $b>\max\left\{\frac{(N-3)u}{2},0\right\}$ and $u_0>Nu$, with $N\geq 2$. Then in the game with N agents the platform attracts all N agents and creates a platform trap iff*

$$u_0<b_0+Nb+\frac{N-1}{2}u$$

and the number of pivotal agents is

$$k_0(N)=\begin{cases} 0 & \text{if} \quad u_0<b_0+(N-1)b+\frac{N-2}{2}u \\ k\in\{1,\dots,N-2\} & \text{if} \quad \begin{array}{c} b_0+(N-1)b+\frac{N+k-3}{2}u\leq u_0 \\ <b_0+(N-1)b+\frac{N+k-2}{2}u \end{array} \\ N-1 & \text{if} \quad \begin{array}{c} b_0+(N-1)b+(N-2)u\leq u_0 \\ <b_0+Nb+\frac{N-1}{2}u \end{array} \end{cases}$$

Proof: The proof is by induction over N. For $N=2$, we know from the results above:

- if $u_0<b_0+b$, then $k_0=0$ and the platform attracts both agents
- if $b_0+b\leq u_0<b_0+2b+\frac{u}{2}$, then $k_0=1$ and the platform attracts both agents.
- if $u_0\geq b_0+2b+\frac{u}{2}$, the platform cannot profitably attract any agents.

Thus, the result holds for $N = 2$.

Now suppose the result holds with $N - 1 \geq 2$ agents and any payoff functions satisfying the assumptions in the text of the proposition. We want to show it also holds with N agents.

First, the induction hypothesis implies that if

$$u_0 < b_0 + (N-1)\, b + \frac{N-2}{2} u,$$

then the platform can attract the last $N - 1$ agents even if the first agent does not join. This implies that on this range, no agent is pivotal, so $k_0(N) = 0$ and the platform can attract all agents with a price $P = b_0 + Nb + (N-1)\, u - u_0 > 0$. This proves the result for N on this parameter range.

Now suppose

$$u_0 \geq b_0 + (N-1)\, b + \frac{N-2}{2} u,$$

so the first agent is pivotal. The game that starts with the second agent after the first agent has joined is a game with $N - 1$ agents and payoff functions $\widetilde{b}(n) = \widetilde{b}_0 + bn$ and $\widetilde{u}(n) = \widetilde{u}_0 - un$, where $\widetilde{b}_0 = b_0 + b > b_0 \geq 0$ and $\widetilde{u}_0 = u_0 - u > (N-1)\, u$. Thus, the payoff functions $\widetilde{b}(n)$ and $\widetilde{u}(n)$ satisfy the assumptions in the text of the proposition with $N - 1$ instead of N. Applying the induction hypothesis, the platform can profitably attract the last $N - 1$ agents conditional on having attracted the first agent iff

$$\widetilde{u}_0 < \widetilde{b}_0 + (N-1)\, b + \frac{N-2}{2} u,$$

which can be rewritten as

$$u_0 < b_0 + Nb + \frac{N}{2} u.$$

And the number of pivotal agents in the game with $N - 1$ agents that starts after the first agent has joined is

$$\widetilde{k}_0(N-1) = \begin{cases} 0 & \text{if} \quad \widetilde{u}_0 < \widetilde{b}_0 + (N-2)\, b + \frac{N-3}{2} u \\ k \in \{1, \ldots, N-3\} & \text{if} \quad \begin{array}{c} \widetilde{b}_0 + (N-2)\, b + \frac{N+k-4}{2} u \leq \widetilde{u}_0 \\ < \widetilde{b}_0 + (N-2)\, b + \frac{N+k-3}{2} u \end{array} \\ N-2 & \text{if} \quad \begin{array}{c} \widetilde{b}_0 + (N-2)\, b + (N-3)\, u \leq \widetilde{u}_0 \\ < \widetilde{b}_0 + (N-1)\, b + \frac{N-2}{2} u \end{array} \end{cases}$$

Plugging in $\widetilde{b}_0 = b_0 + b$ and $\widetilde{u}_0 = u_0 - u$, this can be re-written

$$\widetilde{k}_0(N-1) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + (N-1)\,b + \frac{N-1}{2}u \\ k \in \{1, \ldots, N-3\} & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + \frac{N+k-2}{2}u \leq u_0 \\ < b_0 + (N-1)\,b + \frac{N+k-1}{2}u \end{array} \\ N-2 & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + (N-2)\,u \leq u_0 \\ < b_0 + Nb + \frac{N}{2}u \end{array} \end{cases}.$$

Thus, the number of pivotal agents in the original game with N agents is $k_0\,(N) = 1 + \widetilde{k}_0(N-1)$, so we have

$$k_0\,(N) = \begin{cases} k \in \{1, \ldots, N-2\} & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + \frac{N+k-3}{2}u \leq u_0 \\ < b_0 + (N-1)\,b + \frac{N+k-2}{2}u \end{array} \\ N-1 & \text{if} \quad \begin{array}{c} b_0 + (N-1)\,b + (N-2)\,u \leq u_0 \\ < b_0 + Nb + \frac{N}{2}u \end{array} \end{cases}.$$

When $b_0 + (N-1)\,b + (N-2)\,u \leq u_0 < b_0 + Nb + \frac{N}{2}u$, all agents but the last one are pivotal, so the platform's profit is

$$Nb\,(N) - \sum_{k=1}^{N} u\,(k-1) = Nb_0 + N^2 b + \frac{N\,(N-1)}{2}u - Nu_0,$$

which is positive only if

$$u_0 < b_0 + Nb + \frac{N-1}{2}u.$$

And since $b > \frac{(N-3)}{2}u$, we have

$$b_0 + (N-1)\,b + (N-2)\,u < b_0 + Nb + \frac{N-1}{2}u < b_0 + Nb + \frac{N}{2}u,$$

and thus the platform can profitably attract N agents iff

$$u_0 < b_0 + Nb + \frac{N-1}{2}u.$$

Combined with the expression of $k_0\,(N)$ above, this shows the result holds for N agents. ■

In order to illustrate why the assumption $b > \frac{(N-3)}{2}u$ is crucial to obtain a full closed form characterization of the equilibrium, consider the case with linear payoff functions ($b\,(n) = b_0 + bn$, $u\,(n) = u_0 - un$, with $\min\{b, u\} > 0$) and $N = 4$. Indeed, for $N \leq 3$, the assumption $b > \frac{(N-3)}{2}u$ holds trivially as long as $b > 0$, which we assume.

We can apply the general analysis of the case $N = 3$ above to fully characterize the equilibrium when the payoff functions are linear. The platform can profitably attract all three agents iff

$$u_0 < b_0 + 3b + u.$$

The number of pivotal agents is

$$k_0(3) = \begin{cases} 0 & \text{if} \qquad\qquad u_0 < b_0 + 2b + \frac{u}{2} \\ 1 & \text{if} \quad b_0 + 2b + \frac{u}{2} \le u_0 < b_0 + 2b + u \\ 2 & \text{if} \quad b_0 + 2b + u \le u_0 < b_0 + 3b + u \end{cases}.$$

Now suppose $N = 4$.

If $u_0 < b_0 + 3b + u$, then the platform can attract the last three agents regardless of the participation of the first agent, so $k_0(4) = 0$ and the platform profitably attracts all four agents.

Suppose $u_0 \ge b_0 + 3b + u$, so the first agent with $N = 4$ is pivotal. Once the first agent joins, we can use the characterization with $N = 2$ to conclude that the second agent is non-pivotal iff $b(3) - \frac{u(2)+u(1)}{2} > 0$, i.e., iff

$$u_0 < b_0 + 3b + \frac{3u}{2}.$$

Thus, $k_0(4) = 1$ iff

$$b_0 + 3b + u \le u_0 < b_0 + 3b + \frac{3u}{2},$$

and in this region the platform's profits are

$$4b(4) - u(0) - 3u(3) = 4\left(b_0 + 4b + \frac{9}{4}u - u_0\right) > 0.$$

Next, suppose $u_0 \ge b_0 + 3b + \frac{3}{2}u$, so the first two agents are pivotal. Once the first two agents have joined, the third agent is non-pivotal iff $b(3) - u(2) > 0$, i.e., iff

$$u_0 < b_0 + 3b + 2u.$$

Thus, if

$$b_0 + 3b + \frac{3}{2}u \le u_0 < b_0 + 3b + 2u,$$

then $k_0(4) = 2$ and the platform's profits are

$$4b(4) - u(0) - u(1) - 2u(3) = 4\left(b_0 + 4b + \frac{7u}{4} - u_0\right).$$

Thus, $k_0(4) = 2$ iff

$$b_0 + 3b + \frac{3u}{2} \leq u_0 < \min\left\{b_0 + 4b + \frac{7u}{4}, b_0 + 3b + 2u\right\}.$$

Finally, if $u_0 \geq b_0 + 3b + 2u$, then $k_0(4) = 3$ and platform profits are

$$4\left(b_0 + 4b + \frac{3u}{2} - u_0\right).$$

Thus, $k_0(4) = 3$ is possible only if $b > \frac{u}{2}$, in which case the region where this holds is

$$b_0 + 3b + 2u \leq u_0 < b_0 + 4b + \frac{3u}{2}.$$

Bottom line:

- if $b \leq \frac{u}{4}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 4b + \frac{7u}{4}$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 4b + \frac{7u}{4} \end{cases}$$

- if $\frac{u}{4} < b \leq \frac{u}{2}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 3b + 2u$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 3b + 2u \end{cases}$$

- if $b > \frac{u}{2}$, then the platform can profitably attract all four agents iff $u_0 < b_0 + 4b + \frac{3u}{2}$ and

$$k_0(4) = \begin{cases} 0 & \text{if} \quad u_0 < b_0 + 3b + u \\ 1 & \text{if} \quad b_0 + 3b + u \leq u_0 < b_0 + 3b + \frac{3u}{2} \\ 2 & \text{if} \quad b_0 + 3b + \frac{3u}{2} \leq u_0 < b_0 + 3b + 2u \\ 3 & \text{if} \quad b_0 + 3b + 2u \leq u_0 < b_0 + 4b + \frac{3u}{2} \end{cases}.$$

This illustrates how the number of possible cases and piecemeal segments of the platform's profit function increase very rapidly once N increases beyond 3, even assuming linear payoff functions. It also clarifies why the additional restriction $b > \frac{(N-3)u}{2}$ was imposed in Proposition 11 above: it ensured that for every $N' \leq N$, the number of pivotal agents $k_0(N')$ could take all values between

0 and $N' - 1$, so that the highest value of u_0 for which the platform could profitably attract N' agents always fell in the region where all agents are pivotal, i.e., $k_0(N') = N' - 1$.

F Limited price discrimination

Suppose the platform can change its price only once: it sets P_1 for the first N_1 agents, and then P_2 for the remaining $N_2 = N - N_1$ agents, where P_1 can be different from P_2 if the platform wants. As in the general model, the platform cannot commit to future prices, so P_2 is chosen only when offered. The platform can also choose N_1, i.e., the point at which the price changes.

With this set-up, if $b(N-1) > u(0)$, the platform can replicate the maximal platform trap with full price discrimination from Corollary 1, despite having significantly less price flexibility. The platform can attract the first agent with an offer of $P_1 = \Delta(N) > 0$. Indeed, from Proposition 4, the agent knows that rejecting this offer will lead the platform to set a uniform price $P_2 = b(N-1) - u(0)$ to all remaining agents, which ensures their participation, and therefore a payoff of $u(N-1)$ for the first agent. And if the first agent accepts, it becomes even easier to attract the others, so the first agent obtains $b(N) - P_1 = u(N-1)$. Thus, anticipating that either way the remaining $N-1$ agents will join, the first agent accepts. The same logic applies for all subsequent agents: each of them accepts $P_1 = \Delta(N)$, so ultimately all agents join, just as in Corollary 1. Thus, we have proven:

Proposition 12. *If the platform can change its price only once, but chooses when to do so, and if $b(N-1) > u(0)$ and $b(\cdot)$ is weakly increasing, there exists a unique equilibrium in which all agents join at a uniform price $P_1 = P_2 = \Delta(N) > 0$. All agents would be strictly better off without the platform.*

Comparing this result with Proposition 4 highlights that one of the key ways a platform trap can be generated is a platform's ability to adjust its prices to attract future agents. This ability can arise from the platform having some discretion to adjust its prices (Proposition 12) given we assumed commitment was not possible.

G Private offers

Suppose agents only observe their own offers and nothing else (we call these private offers). Thus, agents do not know their position in the offer sequence or how many agents have previously accepted or rejected offers. As a result, multiple equilibria can arise depending on agents' off-equilibrium path beliefs, and the relevant equilibrium concept in this case is perfect Bayesian equilibrium (PBE).

In order to rule out PBE supported by unreasonable off-equilibrium path beliefs, we make the following refinement. We assume each agent believes the prices the platform will set to subsequent agents are unaffected by its decision to accept or reject its offer. These beliefs are known as passive beliefs in the literature on private contracting (e.g. Rey and Vergé, 2004). In our setting, such beliefs are reasonable. Even if an agent deviates from the proposed equilibrium and rejects an offer, there is no reason to expect the platform to change its subsequent offers since subsequent agents cannot observe this deviation and would therefore be expected to continue to accept the original equilibrium offers, which extract as much as possible from the agents.

Another reason for focusing on equilibrium selection using passive beliefs is that these are also the equilibria that would be selected if we reorder the moves of the players so that all agents make their joining decisions (possibly simultaneously) after the platform has made its private offers to all agents. In that case, the platform cannot change its prices based on agents' decisions. Thus, the PBE we focus on do not depend on the exact ordering of the moves of players (provided, of course, the platform sets each agent's private offer before the agent decides).

The following proposition characterizes the best and the worst such equilibria for the platform, which implies the identical prices and outcomes to Proposition 5 in the main text.

Proposition 13. *Suppose agents observe only their own offers. There exists a continuum of perfect Bayesian equilibria under passive beliefs. These are also the equilibria of the game in which the platform makes its private offers to all agents in the first stage and agents all decide simultaneously whether to accept or not in the second stage.*

1. *In the best equilibrium for the platform, each agent k is charged $P^k = \Delta(N)$, all N agents join, and platform profits are $N\Delta(N)$. All agents would be strictly better off without the platform.*
2. *If $\Delta(1) > 0$, in the worst equilibrium for the platform, each agent k is charged $P^k = \Delta(1)$, all N agents join, and platform profits are $N\Delta(1)$. All agents would be strictly worse off without the platform if $b(N) > b(1)$, indifferent if $b(N) = b(1)$, and would be strictly better off without the platform if $b(N) < b(1)$. If $\Delta(1) \leq 0$, in the worst equilibrium for the platform, no agents join and the platform profits are zero.*

Proof: In the best possible equilibrium for the platform, the platform prices at $P = \Delta(N)$ to all agents, and each agent joins because they believe every other agent faces the same price and also joins. The platform attains its maximum feasible profits and all agents receive net payoff $u(N-1)$, so they would be strictly better off without the platform.

Suppose $\Delta(1) > 0$. The worst equilibrium for the platform is supported by the following strategies and beliefs:

- The platform charges $P^k = \Delta(1)$ to every agent k in the sequence and all agents choose to join
- Provided $P^k \leq \Delta(1)$, each agent believes every other agent is charged $P^k = \Delta(1)$ and they will join, so they also join (indeed, joining yields $b(N) - P \geq b(N) - \Delta(1) \geq u(N-1)$ because $\Delta(\cdot)$ is weakly increasing)
- If an agent is charged a price $P > \Delta(1)$, they believe every other agent is also charged the same P and does not join because they believe no other agent will join either.

Given the agents' strategy, the platform's pricing is optimal to induce participation whenever it is profitable. Conversely, each agent's strategy is optimal given their beliefs, which are fulfilled along the equilibrium path. The resulting platform profit, $N\Delta(1)$, is the minimum profit the platform can obtain given $\Delta(1) > 0$. To see this suppose there is an even worse equilibrium in which the platform sets the price $P^k < \Delta(1)$ to at least one agent. The platform can profitably increase its price P^k to $\Delta(1)$ for any such agent. Then regardless of the number n of other agents this agent expects to join after the price change, it will want to join. Indeed, joining yields payoff $b(n+1) - \Delta(1)$, not joining yields $u(n)$ (recall our PBE refinement implies n does not depend on the agent's own decision), and $b(n+1) - u(n) = \Delta(n+1) \geq \Delta(1)$. Similarly, the platform could change the price of any agent that doesn't join in the proposed equilibrium to $\Delta(1)$ and also profitably induce them to join. Thus, the platform can profitably deviate, so we can rule out any such worse equilibrium.

If $\Delta(1) \leq 0$, the platform cannot profitably attract any agents in the worst equilibrium defined above. Moreover, if it tries to charge a price above $\Delta(1)$ to any agent, then following the beliefs in the worst equilibrium above, such an agent will not join. ■

H Proofs of remaining propositions

This section contains the proofs of the three propositions in Section 7.

H.1 Negotiated prices

Here we prove the results in Section 7.1 (including Proposition 6).

We first show by backwards induction that the price charged to the k-th agent to decide is

$$P^k = (1-\alpha)\left(\sum_{j=0}^{N-k} C_j^{N-k}(1-\alpha)^j \alpha^{N-k-j}\Delta(N'+j+1)\right) \tag{16}$$

and all subsequent agents (including the k-th agent) will participate, where N' is the number of agents that have decided to participate before k.

Consider the last agent to decide whether to participate and suppose N' other agents have already decided to participate. When the last agent decides whether or not to participate, it expects to get $b(N'+1) - P^N$ if it participates and $u(N')$ if it doesn't. Meanwhile, the platform obtains an additional profit of P^N if the agent participates and no additional profit if it doesn't. Given each agent's bargaining power relative to the platform is measured by the parameter α, the Nash bargaining solution is the price P^N that maximizes

$$\left(\Delta(N'+1) - P^N\right)^{\alpha}\left(P^N\right)^{1-\alpha},$$

which implies

$$P^N(N') = (1-\alpha)\Delta(N'+1).$$

Since $\Delta(N'+1) > 0$, the platform will induce the last agent to participate because it can extract a positive price from doing so.

Now suppose the result holds in (16) for all $k \geq K$ for some $K \leq N$. Let us now show it also holds for $k = K-1$. To determine P^{K-1}, suppose N' other agents have already decided to participate. The $(K-1)$-th agent knows regardless of whether it decides to participate, the next $N-K+1$ agents will participate (by the induction hypothesis). Thus, the agent's increase in surplus from participating relative to not participating is

$$\Delta(N'+N-K+2) - P^{K-1}.$$

If the agent participates, the platform's profit from this and the next $N-K+1$ agents participating is

$$\begin{aligned} & P^{K-1} + \sum_{m=0}^{N-K} P^{K+m} \\ = \quad & P^{K-1} + (1-\alpha)\sum_{m=0}^{N-K}\left(\sum_{j=0}^{N-K-m} C_j^{N-K-m}(1-\alpha)^j \alpha^{N-K-m-j}\Delta(N'+m+j+2)\right). \end{aligned}$$

If the $(K-1)$-th agent does not participate, then the platform's profit from the subsequent agents is

$$(1-\alpha)\sum_{m=0}^{N-K}\left(\sum_{j=0}^{N-K-m} C_j^{N-K-m}(1-\alpha)^j \alpha^{N-K-m-j}\Delta(N'+m+1+j)\right).$$

Thus, the Nash bargaining solution is

$$\begin{aligned} P^{K-1} &= (1-\alpha)\,\Delta\left(N'+N-K+2\right) \\ &\quad -\alpha\left(1-\alpha\right)\sum_{m=0}^{N-K}\left(\sum_{j=0}^{N-K-m} C_j^{N-K-m}\left(1-\alpha\right)^j \alpha^{N-K-m-j}\left(\begin{array}{c} \Delta\left(N'+m+j+2\right) \\ -\Delta\left(N'+m+1+j\right)\end{array}\right)\right)\end{aligned}$$

After some re-arrangement of the various terms, this can be re-written

$$P^{K-1} = (1-\alpha)\left(\begin{array}{c} (1-\alpha)^{N-K+1}\Delta\left(N'+N-K+2\right)+\alpha^{N-K+1}\Delta\left(N'+1\right) \\ +\sum_{s=1}^{N-K}\alpha^{N-K+1-s}\Delta\left(N'+s+1\right)\left(\begin{array}{c} \sum_{m=0}^{s} C_{s-m}^{N-K-m}\left(1-\alpha\right)^{s-m} \\ -\sum_{m=0}^{s-1} C_{s-1-m}^{N-K-m}\alpha\left(1-\alpha\right)^{s-1-m}\end{array}\right)\end{array}\right)$$

We wish to show this can be rewritten as

$$P^{K-1} = (1-\alpha)\left(\sum_{s=0}^{N-K+1} C_s^{N-K+1}\left(1-\alpha\right)^s \alpha^{N-K+1-s}\Delta\left(N'+s+1\right)\right),$$

so we need to prove

$$\begin{aligned} &\sum_{m=0}^{s}\frac{(N-K-m)!}{(N-K-s)!\,(s-m)!}\left(1-\alpha\right)^{s-m}-\sum_{m=0}^{s-1}\frac{(N-K-m)!}{(N-K-s+1)!\,(s-1-m)!}\alpha\left(1-\alpha\right)^{s-1-m} \\ = \;& C_s^{N-K+1}\left(1-\alpha\right)^s\end{aligned}$$

for all $s \in \{1,\ldots,N-K\}$ and $\alpha \in [0,1]$, which can be rewritten

$$\sum_{m=0}^{s}\frac{(N-K-s+m)!}{(N-K-s)!m!}\left(1-\alpha\right)^m-\sum_{m=0}^{s-1}\frac{(N-K-s+1+m)!}{(N-K-s+1)!m!}\alpha\left(1-\alpha\right)^m = C_s^{N-K+1}\left(1-\alpha\right)^s.$$

With the change of variable $x = 1-\alpha$, this is equivalent to proving

$$\begin{aligned} &\frac{1}{(N-K-s)!}\sum_{m=0}^{s}\left(N-K-s+m\right)\times\ldots\times\left(1+m\right)x^m \\ &-\frac{1}{(N-K-s+1)!}\sum_{m=0}^{s-1}\left(N-K-s+1+m\right)\times\ldots\times\left(1+m\right)\left(1-x\right)x^m \\ = \;& \frac{(N-K+1)!}{(N-K+1-s)!s!}x^s\end{aligned}$$

for all $s \in \{1,\ldots,N-K\}$ and $x \in [0,1]$.

To do this, let

$$\begin{aligned}
f_s(x) &= \frac{1}{(N-K-s)!}\sum_{m=0}^{s}(N-K-s+m)\times\ldots\times(1+m)\,x^m \\
&\quad -\frac{1}{(N-K-s+1)!}\sum_{m=0}^{s-1}(N-K-s+1+m)\times\ldots\times(1+m)\left(x^m-x^{m+1}\right) \\
g_s(x) &= \frac{(N-K+1)!}{(N-K+1-s)!s!}x^s.
\end{aligned}$$

Note that both $f_s(x)$ and $g_s(x)$ are polynomials in x with the highest power of x being s. We have

$$f_s(0)=0=g_s(0).$$

For all $k\in\{1,\ldots,s-1\}$ we have

$$\begin{aligned}
\frac{d^kf_s}{dx^k}(x=0) &= \frac{(N-K-s+k)!}{(N-K-s)!}-\frac{((N-K-s+1+k)!-(N-K-s+k)!k)}{(N-K-s+1)!} \\
&= \frac{(N-K-s+k)!}{(N-K-s)!}-\frac{(N-K-s+k)!}{(N-K-s)!} \\
&= 0=\frac{d^kg_s}{dx^k}(x=0)
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{d^sf_s}{dx^s}(x=0) &= \frac{(N-K)!}{(N-K-s)!}+\frac{(N-K)!s}{(N-K-s+1)!} \\
&= \frac{(N-K+1)!}{(N-K-s+1)!}=\frac{d^sg_s}{dx^s}(x=0).
\end{aligned}$$

Thus, we can conclude $f_s(x)=g_s(x)$ for all $x\in[0,1]$ and any $s\in\{1,\ldots,N-K\}$.

So by induction we have proven

$$P^k(N')=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(N'+j+1)\right)$$

for any $k\leq N$, when N' agents have decided to participate prior to agent k.

Since all agents participate along the equilibrium path, we can conclude that in equilibrium the price charged to agent k is

$$P^k=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(k+j)\right)$$

as stated in Proposition 6.

Note the price P^k is independent of k in the special case of $\alpha = 0$, so the platform sets the same price to all agents in equilibrium. When $\alpha = 0$, the platform has all the bargaining power, and we have $P^k = \Delta(N)$, so this is identical to Corollary 1 in which the platform makes a take-it-or-leave-it offer to each agent. Also, as $\alpha \to 1$, agents have all the bargaining power, so we have $P^k \to 0$ for all k. Whether agents would be better off without the platform in that case just depends on whether the platform is inefficient or not, i.e. whether $b(N) < u(0)$.

The more interesting case arises when $\alpha > 0$. Since $\sum_{j=0}^{N-k} C_j^{N-k} (1-\alpha)^j \alpha^{N-k-j} = 1$ for any k, the platform's optimal price to the k-th agent is $(1-\alpha)$ multiplied by a weighted average of the surplus obtained from joining the platform for each of the subsequent $N-k+1$ agents (including the k-th agent) deciding. That is, it is a weighted average of the $\Delta(n)$ terms, with n ranging from k to N. We wish to determine whether the platform's price is increasing across agents that sign up and whether agents are better or worse off due to the platform.

Since no agent is pivotal, by assumption, an agent expects all other agents to participate regardless of its decision. Thus, an agent's gross surplus $\Delta(N)$ from joining versus not doesn't change with k. Instead, any change in P^k over k happens due to changes in the additional payoff that the platform stands to gain on subsequent agents joining when agent k joins. Specifically, when agent k joins rather than doesn't, the additional profit that the platform stands to gain beyond the current price can be measured by the extent to which its negotiated price is expected to increase over subsequent agents. As k increases, with fewer potential agents left to join, the impact on the platform's profit from not signing an agent is felt over fewer subsequent agents signing up. This suggests the platform has less to lose from not attracting an agent as k increases, leading to P^k being increasing in k. But the additional profit that can be extracted from subsequent agents also depends on $\Delta(\cdot)$. Given $\Delta(\cdot)$ is an increasing function, as k increases, the average of the additional profit that can be extracted from the remaining agents increases, meaning the platform could actually have more to lose. The overall net effect on how much the platform stands to lose by not attracting an agent as k increases is ambiguous, and depends on the particular shape of $\Delta(\cdot)$.

To proceed, assume linear externalities so $b(n) = b_0 + b_1 n$ and $u(n) = u_0 - u_1 n$ with $u_1 > 0$, which implies $\Delta(n) = \beta_0 + \beta_1 n$, where $\beta_0 = b_0 - u_0 - u_1$ and $\beta_1 = b_1 + u_1$. We must assume $\beta_1 > 0$ so that $\Delta(\cdot)$ is increasing and $\beta_0 + \beta_1 > 0$ so that $\Delta(1) > 0$. Plugging this $\Delta(n)$ function into the last expression of P^k above implies

$$P^k = (1-\alpha)\left(\beta_0 + \beta_1 \left(N - \alpha\left(N-k\right)\right)\right),$$

for all $k \in \{1, \ldots, N\}$, as given in Proposition 6. Clearly, P^k is increasing in k. The monotonicity of P^k in k allows us to characterize when the platform trap arises. Specifically, there are three possible cases:

- If $b(N) - P^N - u(0) > 0$, which is equivalent to
$$\alpha > \frac{u_1(N-1)}{\beta_0 + \beta_1 N},$$
then all agents are better off with the platform.
- If $b(N) - P^1 - u(0) < 0$, then all agents are worse off with the platform. Note that $b(N) - P^1 - u(0) < 0$ is equivalent to
$$b_0 + b_1 N - (1-\alpha)(\beta_0 + \beta_1 N - \alpha\beta_1(N-1)) - u_0 < 0,$$
and it is easily verified that the LHS is increasing in α for $\alpha \leq 1$, equal to $-u_1(N-1)$ when $\alpha = 0$ and $b_0 + b_1 N - u_0$ when $\alpha = 1$.
- If $b(N) - P^1 - u(0) > 0 > b(N) - P^N - u(0)$, then there exists $\overline{k} \in \{1, \ldots, N-1\}$ such that the first $\overline{k}$ agents are better off and the last $N - \overline{k}$ agents are worse off with the platform.

From this, we can conclude:

- If $b_0 + b_1 N - u_0 < 0$, which means $b(N) < u(0)$, then all agents are worse off with the platform regardless of α.
- If $b_0 + b_1 N - u_0 > 0$, then there exist $\underline{\alpha}$ and $\overline{\alpha}$ such that $0 < \underline{\alpha} < \overline{\alpha} < 1$ and
  - if $\alpha \leq \underline{\alpha}$, then all agents are worse off with the platform
  - if $\alpha \geq \overline{\alpha}$, then all agents are better off with the platform
  - if $\underline{\alpha} < \alpha < \overline{\alpha}$, then there exists $\overline{k} \in \{1, \ldots, N-1\}$ such that the first $\overline{k}$ agents are better off and the last $N - \overline{k}$ are worse off with the platform.

H.2 Heterogeneous agents

H.2.1 Proof of Proposition 7

Suppose first $\min\{b(N), b(N+S-1)\} > u(0)$. We show that in this case the platform can profitably attract all agents and extract maximum profits regardless of the order of offers. Suppose the platform has attracted the first N agents. If the last agent is the superstar, the platform attracts it with a price of $b(N+S)-u(N)$ and if it is a regular agent the platform attracts it with a price of $b(N+S)-u(N+S-1)$, both of which are positive by assumption.

Now suppose the platform has attracted the first $N-1$ agents it has made offers to and is now facing the second-to-last agent. We know from the previous case that if this agent participates, then so will the last agent. Now suppose the second-to-last agent does not participate. Then, if the last agent is the superstar, the platform attracts it by charging $b(N+S-1)-u(N-1)>0$, and if it is a regular agent, the platform attracts it by charging $b(N)-u(N-1)>0$ (if the second-to-last agent was the superstar) or $b(N+S-1)-u(N+S-2)>0$ (if the second-to-last agent was not the superstar). Thus, the last agent will be attracted regardless of what the second-to-last agent does, so the platform can attract it by charging $b(N+S)-u(N)$ if it is the superstar, or $b(N+S)-u(N+S-1)$ if it is a regular agent.

We can use the same logic all the way back to the first agent, concluding that the platform profitably attracts all agents, charging $b(N+S)-u(N)$ to the superstar and $b(N+S)-u(N+S-1)$ to regular agents, regardless of the order of offers. The platform cannot do any better because it extracts the maximum surplus from each agent given the participation of the other agents.

Next suppose $b(N+S-1)>u(0)\geq u(N-1)>b(N)$, so the superstar is pivotal but none of the regular agents are. In this case, the only way for the platform to achieve maximum profits is by approaching all regular agents first and the superstar last. All regular agents are offered a price $b(N+S)-u(N+S-1)$ and the superstar is offered a price $b(N+S)-u(N)$, resulting in total profits

$$(N+S)\,b(N+S)-Su(N)-Nu(N+S-1)>0.$$

This can be easily verified as an equilibrium outcome given the same backwards induction logic as above. Once again, the platform attains its absolute maximum profit, except that here the order matters. If the platform approaches the superstar earlier than in the last slot, say in slot $k+1<N+1$, then it would only be able to charge it $b(N+S)-u(k)$ (if the superstar does not participate, the platform will not be able to attract any of the remaining agents), which is strictly less than $b(N+S)-u(N)$.

Finally, suppose $u(N+S-2)>b(N+S-1)$, so all agents are pivotal. Note this implies $u(N-1)>b(N)$ because

$$b(N)-u(N-1)=\Delta(N)\leq\Delta(N+S-1)=b(N+S-1)-u(N+S-2)<0.$$

If the platform approaches the superstar in position $k_0\geq 1$, then it charges $P^k=b(N+S)-u(k-1)$ to regular agents approached in position $k<k_0$ and $P^k=b(N+S)-u(S+k-1)$ to regular agents approached in position $k\geq k_0+1$. The superstar is charged $P^{k_0}=Sb(N+S)-$

$Su\left(k_0-1\right)$. Thus, the platform's profits when it approaches the superstar in position k_0 are

$$\Pi\left(k_0\right)=\left(N+S\right)b\left(N+S\right)-\sum_{k=1}^{k_0-1}u\left(k-1\right)-Su\left(k_0-1\right)-\sum_{k=k_0+1}^{N}u\left(S+k-1\right).$$

Now consider the difference

$$\Pi\left(k_0+1\right)-\Pi\left(k_0\right)=\left(S-1\right)u\left(k_0-1\right)-Su\left(k_0\right)+u\left(S+k_0-1\right).$$

It is easily seen that if $u\left(\cdot\right)$ is convex, then

$$\frac{\left(S-1\right)}{S}u\left(k_0-1\right)+\frac{1}{S}u\left(S+k_0-1\right)\geq u\left(\frac{\left(S-1\right)}{S}\left(k_0-1\right)+\frac{1}{S}\left(S+k_0-1\right)\right)=u\left(k_0\right),$$

so $\Pi\left(k_0+1\right)-\Pi\left(k_0\right)\geq 0$ for all $1\leq k_0\leq N$, which implies $\Pi\left(k_0\right)$ is maximized by $k_0=N$.

And if $u\left(\cdot\right)$ is concave, then

$$\frac{\left(S-1\right)}{S}u\left(k_0-1\right)+\frac{1}{S}u\left(S+k_0-1\right)\leq u\left(k_0\right),$$

so $\Pi\left(k_0+1\right)-\Pi\left(k_0\right)\leq 0$ for all $1\leq k_0\leq N$, which implies $\Pi\left(k_0\right)$ is maximized by $k_0=1$.

H.2.2 Superstar of the same size as regular agents

Now suppose the superstar has the size and therefore the payoffs of an individual agent. In this case, the sole defining characteristic of the superstar is its outsized impact (externality) on all other agents' payoffs — other than that, the superstar is identical to any other agent. In the following proposition, we characterize the outcome for the same three parameter ranges as in Proposition 7.

Proposition 14. *Suppose there are N regular agents and one superstar agent, of size one but equivalent in impact to $S>1$ regular agents.*

1. *If $\min\left\{b\left(N\right),b\left(N+S-1\right)\right\}>u\left(0\right)$, the platform profitably attracts all agents, none are pivotal, and the order of the platform's offers is irrelevant.*
2. *If $b\left(N+S-1\right)>u\left(0\right)>u\left(N-1\right)>b\left(N\right)$, the platform profitably attracts all agents, only the superstar is pivotal, and the platform optimally offers to the superstar last.*
3. *If $b\left(S+N-1\right)<u\left(S+N-2\right)$ and $b\left(N+S\right)>\frac{u(0)+\sum_{k=0}^{N-1}u(S+k)}{N+1}$, the platform profitably attracts all agents, all are pivotal, and the platform optimally offers to the superstar first.*

In cases (1) and (2), all agents would be strictly better off without the platform. In case (3), the regular agents would be strictly better off without the platform and the superstar is indifferent.

Proof of Proposition 14:

The proof of the first two cases is identical to that in Proposition 7 above.

Now suppose $u(N+S-2) > b(N+S-1)$, so all agents are pivotal. Note this implies $u(N-1) > b(N)$ because

$$b(N) - u(N-1) = \Delta(N) \leq \Delta(N+S-1) = b(N+S-1) - u(N+S-2) < 0.$$

In this case, we show that it is optimal for the platform to approach the superstar agent first if it wants to attract any agents. Indeed, by doing so, it can charge $b(N+S) - u(0)$ to the superstar agent and $P^k = b(N+S) - u(S+k-1)$ to agent $k \in \{1, \ldots, N\}$ in the sequence of regular agents. Total profits are

$$(N+1)\, b(N+S) - u(0) - u(S) - u(S+1) - \ldots - u(S+N-1),$$

which is positive if

$$b(N+S) > \frac{u(0) + \sum_{k=0}^{N-1} u(S+k)}{N+1}.$$

Now suppose the platform approaches the superstar second (after one regular agent). Then it charges $b(N+S) - u(0)$ to the first regular agent, $b(N+S) - u(1)$ to the superstar, $b(N+S) - u(S+1)$ to the second regular agent, and so on. Total profits are now

$$(N+1)\, b(N+S) - u(0) - u(1) - u(S+1) - \ldots - u(S+N-1).$$

This is lower than the profit obtained by approaching the superstar first since $u(S) < u(1)$. And it is easily seen that the same will be true when the platform approaches the superstar in any but the first position.

H.3 Competing platforms

Proof of Proposition 8:

There are four cases to consider.

Case (i): $b_1(1,1) > \max\{u(0,0), b_2(0,2)\}$. In this case, platform 1 will profitably attract agent 2 regardless of what agent 1 does. Thus, if agent 1 joins platform 1, it obtains $b_1(2,0) - P_1^1$, if it joins platform 2 it obtains $b_2(1,1) - P_2^1$, and if it doesn't join either platform, it obtains $u(1,0)$. Platform 2 is not willing to offer agent 1 a negative price because it has no chance of attracting agent 2, therefore platform 1 will attract agent 1 because $b_1(2,0) > \max\{u(1,0), b_2(1,1)\}$. And platform 1's prices are

$$P_1^1 = P_1^2 = b_1(2,0) - \max\{u(1,0), b_2(1,1)\} > 0,$$

meaning each agent's net payoff is

$$\max\{u(1,0), b_2(1,1)\}.$$

Thus, both agents are worse off with the platforms if $b_2(1,1) < u(0,0)$ and they are better off with the platforms if $b_2(1,1) > u(0,0)$.

Case (ii): $u(0,0) > b_1(1,1) \geq b_2(0,2)$. In this case, if agent 1 joins platform 1, then platform 1 will also profitably attract agent 2 because $b_1(2,0) \geq b_1(1,1) > b_2(1,1)$ and $b_1(2,0) > u(1,0)$. If agent 1 joins platform 2, then agent 2 will still join platform 1 because $b_1(1,1) \geq b_2(0,2) > u(0,1)$. Finally, if agent 1 does not join either platform, then agent 2 joins platform 1 if $b_1(1,0) > u(0,0)$, and it joins neither platform if $u(0,0) \geq b_1(1,0)$. Thus, platform 2 is unwilling to offer agent 1 a negative price because it has no chance of attracting agent 2. Thus, if agent 1 joins platform 1, its payoff will be $b_1(2,0) - P_1^1$, if it joins platform 2 its payoff is $b_2(1,1) - P_2^1$ and if it joins neither platform, its payoff will be $u(0,0)$ (if $u(0,0) \geq b_1(1,0)$) or $u(1,0)$ (if $b_1(1,0) > u(0,0)$). Since platform 2 is unwilling to offer a negative price, platform 1 will always attract agent 1 by charging

$$P_1^1 = \begin{cases} b_1(2,0) - u(0,0) & \text{if } u(0,0) \geq b_1(1,0) \\ b_1(2,0) - \max\{u(1,0), b_2(1,1)\} & \text{if } u(0,0) < b_1(1,0) \end{cases}$$

Then platform 1 can profitably attract agent 2 by charging

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\} > 0.$$

Platform 1 profits are

$$2b_1(2,0) - u(0,0) - \max\{b_2(1,1), u(1,0)\} \geq 0$$

when $u(0,0) \geq b_1(1,0)$, or

$$2b_1(2,0) - 2\max\{b_2(1,1), u(1,0)\} \geq 0$$

when $u(0,0) < b_1(1,0)$. Note in both cases profits are positive because

$$b_1(2,0) > \max\{b_2(1,1), u(1,0)\}.$$

Agent 1's net payoff is $u(0,0)$ and agent 2's net payoff is $\max\{b_2(1,1), u(1,0)\}$. Since $b_2(1,1) \leq b_2(0,2) < u(0,0)$ in this case, agent 2 is strictly worse off with the platforms. Agent 1 is indifferent.

Case (iii): $b_2(0,2) > b_1(1,1) > u(0,0)$. In this case, if agent 1 joins platform 2, then platform 2 also profitably attracts agent 2. If agent 1 joins platform 1 or neither platform, then platform 1 profitably attracts agent 2 because

$$b_1(2,0) \geq b_1(1,1) > \max\{b_2(1,1), u(0,0)\} \geq \max\{b_2(1,1), u(1,0)\}$$

and $b_1(1,0) > b_2(0,1)$ and $b_1(1,0) \geq b_1(1,1) > u(0,0)$ imply

$$b_1(1,0) > \max\{b_2(0,1), b_1(1,1)\} \geq \max\{b_2(0,1), u(0,0)\}.$$

Thus, since $b_2(0,2) > u(0,0) \geq u(1,0)$, the binding constraint on platform 1 for attracting agent 1 is platform 2. Namely, platform 2 is willing to set its price for agent 1 as low as

$$P_2^1 = -(b_2(0,2) - \max\{b_1(1,1), u(0,1)\}) = -(b_2(0,2) - b_1(1,1)),$$

i.e., to incur a loss on agent 1 equal to the amount platform 2 would be able to extract from agent 2 were it able to attract agent 1. This means platform 1 must set

$$P_1^1 = b_1(2,0) - (2b_2(0,2) - b_1(1,1))$$

in order to attract agent 1, which then implies platform 1 attracts agent 2 by setting

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\}.$$

Total profit for platform 1 is

$$2b_1(2,0) - 2b_2(0,2) + b_1(1,1) - \max\{b_2(1,1), u(1,0)\},$$

which is positive and confirms that platform 1 wins. Net payoffs are $2b_2(0,2) - b_1(1,1)$ for agent 1 and $\max\{b_2(1,1), u(1,0)\}$ for agent 2. Note

$$2b_2(0,2) - b_1(1,1) > u(0,0)$$

in this case, so agent 1 is strictly better off with the platforms. Agent 2 is better off if $b_2(1,1) > u(0,0)$, otherwise it is worse off.

Case (iv): $b_1(1,1) < \min\{b_2(0,2), u(0,0)\}$. In this case, if agent 1 joins either platform, then agent 2 will join the same platform because

$$\begin{aligned} b_2(0,2) &> \max\{b_1(1,1), u(0,1)\} \\ b_1(2,0) &> \max\{b_2(0,2), u(1,0)\} \geq \max\{b_2(1,1), u(1,0)\}. \end{aligned}$$

And if agent 1 joins neither platform, then agent 2 joins neither platform when $u(0,0) \geq b_1(1,0)$, or joins platform 1 when $u(0,0) < b_1(1,0)$. Thus, agent 1's payoffs are $b_1(2,0) - P_1^1$ from joining platform 1, $b_2(0,2) - P_2^1$ from joining platform 2, and $u(0,0)$ or $u(1,0)$ from joining neither platform. Platform 2 is willing to offer agent 1 a negative price by setting

$$P_2^1 = -\left(b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\right),$$

so to win agent 1 platform 1 must set

$$P_1^1 = b_1(2,0) - \max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) \geq b_1(1,0)$ or

$$P_1^1 = b_1(2,0) - \max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) < b_1(1,0)$.

Platform 1 then attracts both agents and charges

$$P_1^2 = b_1(2,0) - \max\{b_2(1,1), u(1,0)\}$$

to agent 2. Total profits for platform 1 are

$$2b_1(2,0) - \max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\} - \max\{b_2(1,1), u(1,0)\}$$

if $u(0,0) \geq b_1(1,0)$ or

$$2b_1(2,0) - \max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\} - \max\{b_2(1,1), u(1,0)\}$$

if $u(0,0) < b_1(1,0)$. It is easily verified that under assumptions (7) and (8), these profits are always positive, so platform 1 does indeed profitably attract both agents. Agent 1's net payoffs are then

$$\max\{u(0,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) > b_1(1,0)$ or

$$\max\{u(1,0), 2b_2(0,2) - \max\{b_1(1,1), u(0,1)\}\}$$

if $u(0,0) \leq b_1(1,0)$. Agent 2's net payoffs are

$$\max\{b_2(1,1), u(1,0)\}.$$

So if $b_2(0,2) > u(0,0) > b_1(1,1)$, then agent 1 is strictly better off with the platforms and agent 2 is strictly worse off.

Based on the four cases laid out above, if $b_2(1,1) > u(0,0)$ (case 1 in the text of the proposition), then we are in cases (i) or (iii), and under this condition, in both cases both agents are better off with the platforms. If $b_1(1,1) > \max\{u(0,0), b_2(0,2)\}$ and $b_2(1,1) < u(0,0)$ (case 2 in the text of the proposition), then we are in case (i) and both agents are worse off with the platforms. Finally, if $b_2(0,2) > \max\{u(0,0), b_1(1,1)\}$ and $b_2(1,1) < u(0,0)$ (case 3 in the proposition), then we are in cases (iii) or (iv), and it is easily verified that agent 1 is better off with the platforms, while agent 2 is worse off.