Online Appendix: Optimal discoverability on platforms

Andrei Hagiu[1] and Julian Wright[2]

A Introduction

In this Online Appendix we consider a number of extensions of the baseline setting which are referred to in the main text.

A.1 Alternative interpretation of x

Suppose x is now interpreted as the probability with which a given seller is exposed to all buyers. Thus, with probability $1-x$ that seller remains only exposed to its initially captive buyers. And symmetrically for the other seller.

The case where neither or only one seller joins is the same as in our baseline model. Consider then the case both sellers join the platform. With probability $(1-x)^2$ all buyers are only exposed to the seller they were initially captive to. In this case, each seller gets $v+b-f-c$. With probability $x(1-x)$ all buyers are exposed to seller 1, which means only seller 2's initially captive buyers learn of it. In this case, seller 1's captives are its own initial captives (who do not know about seller 2) and a fraction of $1-\theta$ of seller 2's captives who are now aware of both products but view them as independent. Meanwhile, seller 2's captives are the fraction $1-\theta$ of its initial captives who view the products as independent (they are all aware of both products). So seller 1's profit is $(1+1-\theta)(v+b-f-c)$ and seller 2's profit is $(1-\theta)(v+b-f-c)$. With the same probability $x(1-x)$, we have the symmetric case in which all buyers are exposed to seller 2, whereas seller 1 only remains exposed to its initially captive buyers. In this case, seller 1's profit is $(1-\theta)(v+b-f-c)$ and seller 2's profit is $(1+(1-\theta))(v+b-f-c)$. Finally, with probability x^2 all buyers are exposed to both sellers, so both sellers get $2(1-\theta)(v+b-f-c)$.

Summing up across these four cases, each seller's expected profit given x is

$$\begin{aligned}
& \left((1-x)^2 + x(1-x)(2-\theta) + (1-x)x(1-\theta) + x^2(2(1-\theta))\right)(v+b-f-c) \\
= \; & (1+x(1-2\theta))(v+b-f-c),
\end{aligned}$$

which is the same as expression (2) in our baseline setting.

[1]Boston University Questrom School of Business. E-mail: ahagiu@bu.edu
[2]Department of Economics, National University of Singapore, E-mail: jwright@nus.edu.sg

The platform's demand when it attracts both sellers is therefore

$$\begin{aligned}\Pi(f,x) &= f\left(2(1-x)^2+2(1+2(1-\theta)+\theta)x(1-x)+(4(1-\theta)+2\theta)x^2\right)\\ &= 2f(1+x(1-\theta)),\end{aligned}$$

which is the same as expression (4).

Thus, this alternative interpretation of x leads to the exact same analysis as the one presented in the paper.

A.2 Price discrimination across buyer groups

Let's suppose the sellers can price discriminate, so each seller can set one price for its own initially captive buyers and a potentially different price for the rival seller's initially captive buyers. This means we can think of there being two separate markets in which the sellers compete: one for seller 1's initially captive buyers and the other for seller 2's initially captive buyers. Each seller's profit will be the sum of its profits from these two markets.

Consider the competition for seller i's initially captive buyers. Seller i will have $\lambda_i = 1-x+x(1-\theta)$ captives among these buyers while seller j will have $\lambda_j = x(1-\theta)$ captives. Moreover, the two sellers would compete over the measure $\phi = x\theta$ of seller i's buyers who are exposed to both sellers and view them as identical substitutes. Given the competing sellers have different measures of captives in this market, the more aggressive seller's profit (the one with fewer captives) is no longer just equal to its profits from captives. The sellers' expected profits can be obtained by applying Proposition 1 from Myatt and Ronayne (2023).[3] We can conclude that seller i's expected profit in this market is

$$(1-x+x(1-\theta))(v+b-f-c) = (1-x\theta)(v+b-f-c),$$

while seller j's expected profit in this market is

$$\left(\frac{x(1-\theta)+x\theta}{1-x+x(1-\theta)+x\theta}\right)(1-x\theta)(v+b-f-c) = x(1-x\theta)(v+b-f-c).$$

The same logic applies in the market for seller j's buyers, and we obtain a symmetric result. Therefore adding the expected profit from the two markets together, each seller's

[3] For the reader's convenience, we have summarized the relevant results from Myatt and Ronayne (2023) in Online Appendix A.5 below.

expected profit from participating on the platform is

$$(1-x\theta)(1+x)(v+b-f-c).$$

This is higher than each seller's profit in the case without price discrimination, which was

$$(1+x-2x\theta)(v+b-f-c).$$

Indeed, each seller's expected profit is higher by $\theta x(1-x)(v+b-c-f)$ as a result of price discrimination, reflecting the fact price discrimination makes them asymmetric competitors in their respective markets. This suggests that an intermediate level of discovery (i.e. $x=\frac{1}{2}$) will lead to the largest increase in the sellers' profits from price discrimination, while when $x=0$ or $x=1$ price discrimination will have no effect on their profits. Note also that a seller's profit from participating on the platform is now single-peaked in x: increasing in x if $x \leq \frac{1-\theta}{2\theta}$ and decreasing in x if $x \geq \frac{1-\theta}{2\theta}$.

For both sellers joining the platform to be an equilibrium, we need to have

$$f \leq b+\frac{(1-x\theta)(1+x)-1}{(1-x\theta)(1+x)}(v-c),$$

so the platform's profit when both sellers join is

$$\begin{aligned} \Pi &= 2\lambda f(1+x(1-\theta)) \\ &= 2(v-c)\left(1+\mu-\frac{1}{(1-x\theta)(1+x)}\right)(1+x(1-\theta)). \end{aligned}$$

After dropping the constant $2(v-c)$, the derivative with respect to x is

$$\frac{d\Pi_{pd}}{dx}=(1+\mu)(1-\theta)-\frac{x\theta(2+(1-\theta)x)}{(1-x\theta)^2(1+x)^2}.$$

By analyzing this, we can characterize x^*. First note $\frac{d\Pi_{pd}}{dx}>0$ when $\theta=0$, implying $x^*=1$ if $\theta=0$. By continuity, this derivative remains strictly positive for θ close enough to zero, so $x^*=1$ for θ below some positive threshold. Also note that if $x=0$, then for any $\theta<1$, $\frac{d\Pi_{pd}}{dx}>0$, implying $x^*>0$ for $0\leq\theta<1$. And $\frac{d\Pi_{pd}}{dx}=0$ iff $\theta=1$, implying $x^*=0$ iff $\theta=1$. Next we show the solution $\widetilde{x}$ to the FOC $\frac{d\Pi_{pd}}{dx}=0$ is decreasing in θ. Totally differentiating the FOC with respect to θ and x implies

$$-\left(1+\mu+\left(\frac{x\left(2+x+\theta x^2\right)}{(1-x\theta)^3(1+x)^2}\right)\right)d\theta-\left(\frac{2\theta\left(1+3x^2\theta+x^3\theta(1-\theta)\right)}{(1-x\theta)^3(1+x)^3}\right)dx=0$$

so

$$\frac{dx}{d\theta}=-\frac{1+\mu+\left(\frac{x\left(2+x+\theta x^2\right)}{(1-x\theta)^3(1+x)^2}\right)}{\frac{2\theta(1+3x^2\theta+x^3\theta(1-\theta))}{(1-x\theta)^3(1+x)^3}}<0$$

implying $\widetilde{x}$ is strictly decreasing in θ. Taken with the results already stated, this implies there is a cutoff $\theta_1(\mu)$, such that $x^*=1$ for $0\leq\theta\leq\theta_1(\mu)$, and $0<x^*<1$ is strictly decreasing in θ for $\theta_1(\mu)<\theta<1$ reaching $x^*=0$ when $\theta=1$. This also implies $\theta_1(\mu)$ is defined by the point where $\frac{d\Pi_{pd}}{dx}=0$ when $x=1$. I.e. $\theta_1(\mu)$ is defined by

$$\frac{\theta}{(1-\theta)^3}(3-\theta)=4(\mu+1),$$

where the LHS is increasing in θ. Comparing this to the threshold level of θ

$$\frac{\theta}{(1-\theta)^3}=4(\mu+1)$$

with uniform pricing, this means $\theta_1(\mu)$ is now lower (closer to zero). Putting these results together with our characterization in the uniform pricing case implies the x^* curve here crosses over the curve for x^* in the uniform pricing case.

To summarize

$$x^*=\begin{cases}1 & if \quad 0<\theta\leq\theta_1(\mu)\\ \widetilde{x} & if \quad \theta_1(\mu)<\theta<1\\ 0 & if \quad \theta=1\end{cases},$$

where

$$\mu=\frac{b}{v-c}$$

and $\theta_1(\mu)\in(0,1)$ is the unique solution in θ to

$$\frac{\theta}{(1-\theta)^3}(3-\theta)=4(\mu+1)$$

and $0<\widetilde{x}<1$ is given by the unique solution to

$$(1+\mu)(1-\theta)=\frac{x\theta(2+(1-\theta)x)}{(1-x\theta)^2(1+x)^2}$$

in the range $\theta_1(\mu)<\theta<1$.

A.3 Commitment

Consider the baseline setting but assume the platform cannot commit to its choice of x. The timing is: the platform chooses its fees, sellers decide whether to join or not, and then the platform decides how much discoverability to induce. Given the platform's fee and with both sellers participating, and assuming $\theta < 1$, the platform always does best by setting maximum discoverability ($x = 1$) since that expands the number of transactions which it earns its fee on—this is easily seen from the expression of platform profits (4) in the baseline.

Given sellers expect $x = 1$, it is an equilibrium for both to join the platform iff

$$2\left(v + b - f - c\right)\left(1 - \theta\right) \geq \left(v - c\right).$$

Taking this constraint into account, the platform will set $f = \max\left\{v + b - c - \frac{v-c}{2(1-\theta)}, 0\right\}$, and so will obtain a profit of

$$\Pi^{*} = 2\lambda\left(b + \left(v - c\right)\frac{\left(1 - 2\theta\right)}{1 + \left(1 - 2\theta\right)}\right)\left(2 - \theta\right)$$

if $\theta < 1 - \frac{v-c}{2(v-c+b)}$, and otherwise will obtain zero profit.

Compared to the platform's profit in the baseline, this is the same in the range of $0 < \theta \leq \theta_1\left(\mu\right)$ where $x^{*} = 1$ in the baseline, and otherwise is lower since its choice of x will no longer be ex-ante profit maximizing.

A.4 Leakage and unfavorable beliefs

Throughout the paper, we have assumed that once a seller decides to join the platform in period 2, it abandons its direct channel, so that in period 3 there is no possibility of leakage, where buyers from the platform switch to purchase from the sellers' direct channels if it is cheaper to do so. Suppose instead that each seller maintains its direct channel even after joining the platform and that in period 3 every seller that joins the platform can set different prices for buyers to purchase via the platform or via the direct channel. All buyers that are aware of a seller observe its prices in both channels. In this case, if $f > b$, then there would be no sales conducted via the platform in period 3. Indeed, a seller that made positive sales via the platform could increase profits by setting the price in its direct channel at the level it is currently charging on the platform and increasing its price on the platform to ensure all its sales are in its direct channel (at the same price but a strictly lower cost). Thus, if the platform sets $f > b$, it makes no profits, and so it must set $f \leq b$.

Similarly, in the baseline setting, and for most of the paper, we have assumed favorable beliefs. This means that for any given fee f, if there is an equilibrium where all sellers join, this equilibrium is selected. Consider instead unfavorable beliefs. This means that for any given fee f, if there is an equilibrium where no sellers join, that equilibrium is selected. The platform will therefore have to set its fee with this equilibrium selection in mind, to make sure for the given fee that each seller wants to join even if the other does not. Thus, the platform can never set $f > b$, otherwise there is always an equilibrium in which no seller joins and that equilibrium prevails. Which means that once again, the platform faces the constraint $f \leq b$.

Consequently, with either costless leakage or unfavorable beliefs, we have to add an additional constraint to the platform's optimization problem, namely $f \leq b$. This means that in the baseline setting, the maximum fee that the platform can charge is

$$f = \min \left\{ b + (v - c) \frac{x (1 - 2\theta)}{1 + x (1 - 2\theta)}, b \right\}.$$

If $\theta \leq \frac{1}{2}$, then the maximum fee that the platform can charge to get both sellers to join is $f = b$. The platform's profit is then $2\lambda b (1 + x (1 - \theta))$ (from expression 4), which is maximized by setting $x = 1$.

If $\theta > \frac{1}{2}$, then the maximum fee the platform can charge to get both sellers to join is

$$f = b + (v - c) \frac{x (1 - 2\theta)}{1 + x (1 - 2\theta)} < b,$$

which is the same as in the baseline model. Platform profits are therefore the same as in the baseline, and so will be the optimal x^*.

Thus, in all cases the optimal level of discoverability x^* is the same, regardless of whether sellers maintain their direct channels and can induce leakage or not, and regardless of whether expectations are favorable or unfavorable. For $\theta > \frac{1}{2}$, f^* and Π^* are also unchanged, whereas for $\theta \geq \frac{1}{2}$ we now obtain

$$\begin{aligned} f^* &= b \\ \Pi^* &= 2\lambda b (2 - \theta). \end{aligned}$$

A.5 Useful results on mixed strategy pricing

We will make use of the following results on mixed strategy equilibrium pricing from Proposition 1 in Myatt and Ronayne (2023).

Suppose there are n identical sellers and measure one of buyers who want to buy one unit of product from one of the sellers which gives them value v. There are $n+1$ types of buyers: λ_i are informed of seller i only for $i = 1, 2..., n$ and $\phi = 1 - \sum_{i=1}^{n} \lambda_i$ are informed of all sellers. Assume $\lambda_i > 0$ for all i, and $\phi > 0$. Seller i has marginal cost $c_i < v$.

Define the critical price p_i^+ such that

$$(\lambda_i + \phi)\left(p_i^+ - c_i\right) = \lambda_i (v - c_i).$$

If a seller sold to all buyers that are informed of its product at this price, it would earn the same as setting the monopoly price and just selling to its captive buyers. Therefore, $c_i < p_i^+ < v$ for all i.

Suppose sellers are ordered so $p_1^+ \geq p_2^+ \geq ... \geq p_n^+$. Seller n is considered the most aggressive seller (it has a combination of a low marginal cost and/or a low number of captives). Then in their Proposition 1, Myatt and Ronayne show that there is a mixed strategy equilibrium where seller n obtains expected profit

$$(\lambda_n + \phi)\left(p_{n-1}^+ - c_n\right)$$

and all other sellers obtain

$$\lambda_i (v - c_i).$$

Note this most aggressive seller obtains greater expected profit than it would obtain by setting $p_n = v$. This is because

$$(\lambda_n + \phi)\left(p_{n-1}^+ - c_n\right) \geq (\lambda_n + \phi)\left(p_n^+ - c_n\right) = \lambda_n (v - c_n),$$

with the inequality in the equation above being strict if $p_{n-1}^+ > p_n^+$.

A few special cases follow:

1. If the sellers are all symmetric because $\lambda_i = \lambda$ for all i and $c_i = c$ for all i, then this equilibrium still applies, and seller n in this case obtains $(\lambda + \phi)\left(p_{n-1}^+ - c\right) = (\lambda + \phi)(p_n^+ - c) = \lambda (v - c)$, so all sellers obtain the same expected profit $\lambda (v - c)$.

2. If there are just two sellers with $\lambda_1 = \lambda_2 = \lambda$ and $c_1 > c_2$ so $p_1^+ > p_2^+$, then seller 2's expected profit is

$$(\lambda + \phi)\left(p_1^+ - c_2\right) = \phi (c_1 - c_2) + \lambda (v - c_2) > \lambda (v - c_2),$$

while seller 1's expected profit is $\lambda (v - c_1)$.

3. If there are just two sellers with $\lambda_1 > \lambda_2$ and $c_1 = c_2 = c$, so $p_1^+ > p_2^+$, then seller 2's expected profit is

$$(\lambda_2 + \phi)\left(p_1^+ - c\right) = \frac{\lambda_2 + \phi}{\lambda_1 + \phi}\lambda_1 (v - c) > \lambda_2 (v - c),$$

while seller 1's expected profit is $\lambda_1 (v - c)$.

4. If there are n sellers such that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ but all with the same marginal cost c, then seller n makes

$$\frac{\lambda_n + \phi}{\lambda_{n-1} + \phi}\lambda_{n-1} (v - c)$$

and all sellers $i \leq n - 1$ make

$$\lambda_i (v - c).$$

A.6 Outside option includes benefits

So far we have assumed that the outside option does not involve any transaction benefits (i.e. tools) for sellers. Suppose instead the outside option is actually a competitive fringe of B2B providers that offer some tools for sellers with transaction benefit b_0. In other words, a seller that does not join the platform makes profits $v - c + b_0$ and let $\mu_0 = \frac{b_0}{v-c}$. We can generalize the results from Proposition 1 to this case.

Proposition *Suppose the platform offers sellers transaction benefit b, while the outside option offers sellers transaction benefit b_0. The platform is viable iff*

$$\frac{1 + \mu_0}{1 + \mu} \leq \max\{2(1 - \theta), 1\}$$

and the platform's optimal choice of discoverability is

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu, \mu_0) \\ \frac{1 - \sqrt{\frac{\theta(1+\mu_0)}{(1-\theta)(1+\mu)}}}{2\theta - 1} & if \quad \theta_1(\mu, \mu_0) \leq \theta \leq \frac{\mu+1}{\mu+2+\mu_0} \\ 0 & if \quad \theta \geq \frac{\mu+1}{\mu+2+\mu_0} \end{cases},$$

where $\theta_1(\mu, \mu_0)$ is the unique solution in θ to

$$\frac{\theta}{(1 - \theta)^3} = \frac{4(1 + \mu)}{1 + \mu_0}.$$

The platform's maximum attainable profits are

$$2\lambda\left(v-c\right)\left(1+\left(1-\theta\right)x^{*}\right)\left(1+\mu-\frac{1+\mu_{0}}{x^{*}\left(1-2\theta\right)+1}\right).$$

The viability of the platform (i.e. its ability to make profit) depends on μ_0 not being too high. When $\theta > \frac{1}{2}$, so with relatively high substitutability, the viability of the platform depends on it offering tools that are at least equal to the outside fringe supplying B2B tools. On the other hand, when $\theta < \frac{1}{2}$, the platform may be viable even when its tools are worse than those offered by the outside fringe. Specifically, this happens when

$$1<\frac{1+\mu_{0}}{1+\mu}\leq 2\left(1-\theta\right).$$

In this range, the platform's ability to enable discoverability allows it to create more value relative to its inferior tools.

Proposition A.6 shows that the optimal level of discoverability is decreasing in the value offered by the outside fringes' tools (as captured by μ_0). This implies that as the generic B2B tools offered to sellers in the market get better, and assuming the platforms' tools don't equally improve (perhaps because the generic B2B tool providers are catching up), platforms will choose less discoverability (assuming they remain viable).

Proof of Proposition A.6:

A seller that does not join the platform makes profits $v + b_0 - c$. A seller that joins the platform alone makes profits$v + b - f - c$. If both sellers join the platform, then each seller makes profits

$$\left(v+b-f-c\right)\left(1-x+2x\left(1-\theta\right)\right).$$

For both sellers joining the platform to be an equilibrium, we need

$$\left(v+b-f-c\right)\left(1-x+2x\left(1-\theta\right)\right)\geq\left(v-c+b_{0}\right),$$

which is equivalent to

$$\left(v-c\right)x\left(1-2\theta\right)-b_{0}\geq\left(f-b\right)\left(1+x\left(1-2\theta\right)\right)$$

Then, assuming favorable expectations, sellers will both join iff

$$\begin{aligned} f &\leq \frac{(v-c+b)\,x\,(1-2\theta)+b-b_0}{x\,(1-2\theta)+1} \\ f &\leq v-c+b-\frac{v-c+b_0}{x\,(1-2\theta)+1} \end{aligned}$$

For the platform, attracting both sellers dominates attracting only one of them, since it can always set $x=0$ and double the profits from attracting just one seller. The platform's maximum attainable profits with two sellers are

$$2\,(v-c)\,(1+(1-\theta)\,x)\left(1+\mu-\frac{1+\mu_0}{x\,(1-2\theta)+1}\right),$$

where

$$\begin{aligned} \mu &= \frac{b}{v-c} \\ \mu_0 &= \frac{b_0}{v-c}. \end{aligned}$$

If $\theta \leq \frac{1}{2}$, then the expression above is increasing in x, so it is maximized for $x^* = 1$. In this case, the platform's maximum profits are non-negative iff

$$\frac{1+\mu_0}{1+\mu} \leq 2\,(1-\theta)\,.$$

Otherwise, the platform is not viable. Note in particular we can have

$$1 < \frac{1+\mu_0}{1+\mu} \leq 2\,(1-\theta)\,,$$

so the platform can be viable even if the outside option offers more value added than the platform. The platform's viability comes from its ability to offer discovery.

Suppose $\theta > \frac{1}{2}$. In this case, $1+\mu-\frac{1+\mu_0}{x(1-2\theta)+1}$ is decreasing in x, so the platform is viable (i.e. can obtain non-negative profits) iff $\mu \geq \mu_0$. So now the platform can no longer be viable if the outside option offers more value added. Then the derivative of the platform's profit expression with respect to x is

$$2\,(v-c)\left((1+\mu)\,(1-\theta)-\frac{\theta\,(1+\mu_0)}{(x\,(1-2\theta)+1)^2}\right),$$

so SOC holds. The solution is

$$x^* = \begin{cases} 1 & if \quad 0 < \theta \leq \theta_1(\mu, \mu_0) \\ \frac{1 - \sqrt{\frac{\theta(1+\mu_0)}{(1-\theta)(1+\mu)}}}{2\theta - 1} & if \quad \theta_1(\mu, \mu_0) \leq \theta \leq \theta_2(\mu, \mu_0) \\ 0 & if \quad \theta \geq \theta_2(\mu, \mu_0) \end{cases},$$

where $\theta_1(\mu, \mu_0)$ is the unique solution in θ to

$$4\frac{1+\mu}{1+\mu_0} \geq \frac{\theta}{(1-\theta)^3}$$

and

$$\theta_2(\mu, \mu_0) = \frac{1}{1 + \frac{1+\mu_0}{1+\mu}}.$$

A.7 Representative buyer specification

Suppose buyers have utility from two goods, which are provided by two different sellers. We adopt a standard quadratic specification:

$$U(q_1, q_2) = v(q_1 + q_2) - (1-\theta)\left(q_1^2 + q_2^2\right) - \frac{\theta}{2}(q_1 + q_2)^2,$$

where θ represents the degree of substitutability between the products, so $\theta = 0$ means goods are independent and $\theta \to 1$ captures goods being homogenous (highly substitutable).

When buyers can choose both sellers

$$\begin{aligned} q_1 &= \frac{v - p_1}{2} - \frac{\theta(p_1 - p_2)}{4(1-\theta)} \\ q_2 &= \frac{v - p_2}{2} - \frac{\theta(p_2 - p_1)}{4(1-\theta)}. \end{aligned}$$

And when they can just choose seller i because that is the only one they are informed of, then $q_j = 0$, and their demand for seller i is given by

$$q_i = \left(\frac{1}{2-\theta}\right)(v - p_i).$$

Note for the same prices ($p_1 = p_2 = p$), a buyer's demand expands from $q_1 = \left(\frac{1}{2-\theta}\right)(v-p)$ to $q_1 + q_2 = (v - p)$ in case the buyer is informed of both sellers rather than just one. Note for $\theta = 1$, there is no demand expansion, while when $\theta = 0$, so the products are independent (fully differentiated), demand expansion is 100%.

Consider seller i without a platform. It comes with an initial measure one of buyers, all of whom only know about it but not its rival, and so its profit is

$$\pi_i = (p_i - c)\left(\frac{1}{2-\theta}\right)(v - p_i).$$

This is maximized by setting the monopoly price $p_i = \frac{v+c}{2}$, leading to a profit of

$$\pi_i = \left(\frac{1}{2-\theta}\right)\left(\frac{v-c}{2}\right)^2.$$

One difference with the baseline setting is that the profit a seller gets when it doesn't participate still depends on θ, since the total demand is less when the goods are fully differentiated but only one is available.

Now suppose only one seller joins the platform. Since the platform just provides a transaction benefit to sellers, so their marginal cost becomes $c - b$, but charges a per-unit fee f, and doesn't help expand the number of customers for the seller, it will get a profit of

$$\pi_i = \left(\frac{1}{2-\theta}\right)\left(\frac{v+b-f-c}{2}\right)^2,$$

and a seller will therefore join the platform if it expects the other seller not to iff $f \leq b$.

Suppose when both sellers join, the platform informs a fraction x of each seller's participating customers about the existence of the rival seller. Consider when each seller would join given the expectation that the other will also join. Each seller faces a fraction $1 - x$ of its initial customers who are only informed of itself and x who are also informed of the other seller. It also enjoys being able to sell to a fraction x of the other seller's initially captive buyers.

Demand for seller i is made up of three components:

- seller i captives that remain captive, which generate demand $(1-x)\left(\frac{1}{2-\theta}\right)(v-p_i)$
- seller i captives that are now buying from both sellers, which generate demand $x\left(\frac{v-p_i}{2} - \frac{\theta(p_i-p_j)}{4(1-\theta)}\right)$
- seller j captives that are now buying from both sellers, which generate demand $x\left(\frac{v-p_i}{2} - \frac{\theta(p_i-p_j)}{4(1-\theta)}\right)$

So seller i's profit is

$$\pi_i = (p_i + b - c - f)\left((1-x)\left(\frac{1}{2-\theta}\right)(v-p_i) + 2x\left(\frac{v-p_i}{2} - \frac{\theta(p_i-p_j)}{4(1-\theta)}\right)\right).$$

From the FOCs, the equilibrium prices are:

$$p^* = p_1 = p_2 = \frac{2\left(x + \frac{1}{1-\theta}\right)(v + c - (b - f)) + \frac{\theta(2-\theta)}{(1-\theta)^2}x\,(c - (b - f))}{4\left(x + \frac{1}{1-\theta}\right) + x\frac{\theta(2-\theta)}{(1-\theta)^2}}.$$

Note the equilibrium price is always decreasing in x when $\theta > 0$ and provided $f < v - c + b$ as we would expect. We also need $v > p^*$, which is easy to confirm given $v > c$. A seller's equilibrium profit when it joins is

$$\pi^* = 2\left(\frac{1-\theta}{2-\theta}\right)(1 + x(1-\theta))^2 \frac{(2(1 + x - \theta) - x\theta(2-\theta))}{(4(1 + x - \theta) - 3x\theta(2-\theta))^2}(v + b - f - c)^2.$$

Let $b = \mu(v - c)$. So we can determine a cutoff level f^c such that the seller's profit is higher when it joins than when it doesn't provided $f \leq f^c$, assuming the other seller joins. This is determined by

$$f^c = (1 + \mu - y(x, \theta))(v - c)$$

where

$$y(x,\theta) = \frac{4(1 + x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}}{2(1 + x(1-\theta))\sqrt{4(1 + x(1-\theta)) + 2x\frac{\theta(2-\theta)}{(1-\theta)}}}.$$

Thus, there is an equilibrium where both sellers join provided $f \leq f^c$. Here f^c can be greater or less than b.

Note $y(0,\theta) = 1$. The platform will set f and x such that both sellers join and the platform's resulting profit is

$$\begin{aligned}
\Pi(x, f) &= f\left(2(1-x)\left(\frac{1}{2-\theta}\right) + 2x\right)(v - p^*) \\
&= 2\frac{(1 + x(1-\theta))}{2-\theta}\frac{2(1 + x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}}{4(1 + x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}} f((1+\mu)(v - c) - f).
\end{aligned}$$

The platform's problem is to set f and x to maximize $\Pi(x, f)$ subject to

$$0 \leq f \leq (1 + \mu - y(x,\theta))(v - c).$$

Note the unconstrained optimal fee equals

$$f^u = (1+\mu)\frac{v - c}{2}.$$

Then define $f^* = \min\{f^c, f^u\}$. Then can normalize $v - c = 1$ since that just scales up

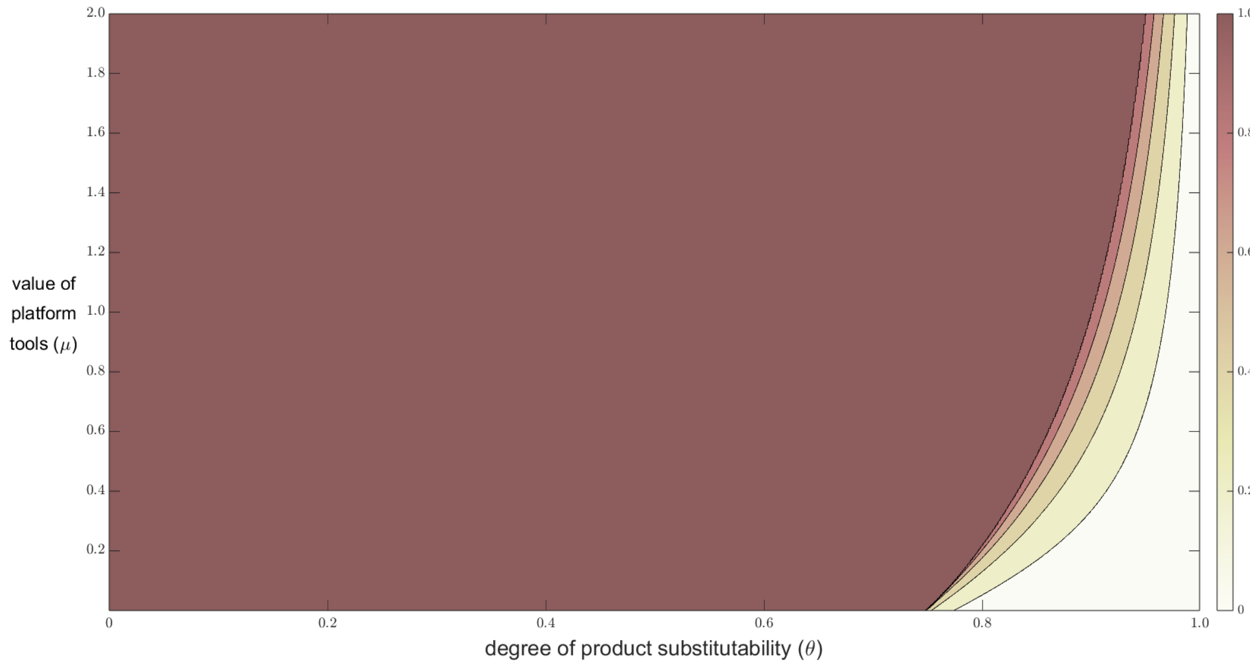

Figure 4: Heatmap of platform's optimal level of discoverability x^*
(darker color indicates higher x^*)

the platform's profit but doesn't affect the optimal choice of x. So the problem becomes to choose $x \in [0,1]$ to maximize

$$\Pi(x) = 2\lambda \frac{(1+x(1-\theta))}{2-\theta} \frac{2(1+x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}}{4(1+x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}} f^* \left((1+\mu) - f^*\right)$$

where

$$f^* = \min\left\{\frac{1+\mu}{2}, 1+\mu - \frac{4(1+x(1-\theta)) + x\frac{\theta(2-\theta)}{(1-\theta)}}{2(1+x(1-\theta))\sqrt{4(1+x(1-\theta)) + 2x\frac{\theta(2-\theta)}{(1-\theta)}}}\right\}.$$

Note it will never be optimal to set x such that $f^* < 0$ because that would yield negative profits and the platform can always guarantee positive profits by setting $x = 0$.

Unlike our baseline setting, there is no longer a closed-form solution for x^*. But in Figure 4 we have mapped out the optimal x^* where θ is on the horizontal axis and μ is on the vertical axis. The figure shows levels of x^* from $x^* = 0$ (lightest colour) to $x^* = 1$ (darkest colour), and it is qualitatively similar to the corresponding figure for the baseline setting (Figure 1 in the main paper).

A.8 Two-part tariffs

We are interested in studying whether the platform would be better off charging sellers a fixed fee instead of, or in addition to, variable transaction fees. To model this, suppose that in addition to the transaction fee f, the platform can also charge each seller a fixed fee F. The remainder of the baseline model setup remains unchanged.

The sellers' payoffs are as in the baseline model except if they join the platform they also pay the fixed fee F. Thus, for both sellers joining the platform to be an equilibrium, we must have

$$F + (f - b)(1 + x(1 - 2\theta)) \leq (v - c)x(1 - 2\theta) \tag{21}$$

and the platform's profit with both sellers joining is

$$2F + 2f(1 + (1 - \theta)x). \tag{22}$$

The platform maximizes (22) over (F, f, x) subject to the constraint (21) above. It is easily seen that the constraint must be binding. Using that to write F as a function of f and x, we obtain that the platform maximizes

$$2((v - c)x(1 - 2\theta) + b(1 + x(1 - 2\theta)) + f\theta x)$$

with respect to f and x. Clearly, the last expression is increasing in f, so the platform will set

$$f^* = v - c + b,$$

which then leads to

$$x^* = 1.$$

This implies

$$\begin{aligned} F^* &= (v - c)x^*(1 - 2\theta) - (f^* - b)(1 + x^*(1 - 2\theta)) \\ &= -(v - c). \end{aligned}$$

Thus, with unrestricted two-part tariffs, the platform chooses maximum discoverability and charges the maximum transaction fee. It extracts the entire margin of each seller's product, and subsidizes the participation of sellers by paying each seller the value of their outside option, which is equal to $v - c$.

At first glance, one may think that with inelastic demand both transaction fees and fixed fees work like transfers, so it should not matter which is used by the platform. However, this is not the case, because an increase in f is just passed through by the sellers to the extent they compete, so it doesn't impact their profit as much as an equivalent increase in a fixed fee that generates the same revenue for the platform. Specifically, in our model, each seller's net profit only reflects transactions with buyers for whom it doesn't compete (initially captive buyers and buyers who view the two sellers' products as independent), whereas the platform derives the transaction fee f from all transactions. Thus, the two sellers do not internalize all transactions they generate on the platform when making their participation decisions, which is why, provided there is some discovery, it always makes sense for the platform to load up on the transaction fee and offset it with a fixed subsidy to the maximum extent possible.

A problem with the solution above is that it involves the platform paying each seller their outside option as a fixed subsidy upfront. The sellers then derive zero net revenues from their participation. In practice, this is unrealistic since it would lead to moral hazard problems (e.g. sellers participate just to collect the subsidy but then not doing anything to serve buyers) and the platform may also face a budget constraint. So it is reasonable to assume that the subsidy the platform can offer to sellers is limited by some exogenously given amount $K \geq 0$, which means we have the additional constraint

$$F \geq -K. \tag{23}$$

The platform maximizes (22) over (F, f, x) subject to the constraint (21) above and the additional constraint (23).

Since platform profits are increasing in F and f, we must have

$$F + (f - b)(1 + x(1 - 2\theta)) = (v - c)\, x(1 - 2\theta).$$

Using this to replace F in the platform's profits and (23), the problem becomes to choose f and x to maximize

$$2\left((v - c + b)\, x(1 - 2\theta) + b + f\theta x\right)$$

subject to

$$(v - c)\, x(1 - 2\theta) - (f - b)(1 + x(1 - 2\theta)) \geq -K.$$

If the constraint is not binding, then $f^* = v - c + b$ and $x^* = 1$, which is valid iff $v - c \leq K$. So assume $0 \leq K < v - c$, and the constraint is binding. Solving the binding constraint for f implies

$$f = (v - c)\frac{x(1 - 2\theta)}{1 + x(1 - 2\theta)} + \frac{K}{1 + x(1 - 2\theta)} + b.$$

Substituting this into the platform's profit, after factoring out the constant $2(v-c)$, the problem is to choose x to maximize

$$\frac{x(1-2\theta)(1+x(1-\theta))}{1+x(1-2\theta)}+\frac{\theta xK}{(v-c)(1+x(1-2\theta))}+\mu(1+x(1-\theta)).$$

It is easily verified that if $\theta\leq\frac{1}{2}$, this is increasing in x, so the platform sets $x^*=1$ regardless of K. Assume therefore $\theta>\frac{1}{2}$. The derivative in x is

$$\frac{(1-2\theta)\left(1+2x(1-\theta)+x^2(1-\theta)(1-2\theta)\right)+\mu(1-\theta)(1+x(1-2\theta))^2+\frac{\theta K}{v-c}}{(1+x(1-2\theta))^2},$$

which is decreasing in x for $\theta>\frac{1}{2}$, so the SOC holds. This derivative is zero when the numerator equals zero, which gives the unconstrained solution

$$x(\theta)=\frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}\left(1-\frac{K}{v-c}\right)}}{2\theta-1}.$$

This is the same as $x(\theta)$ in the baseline except $\mu+1>1$ is replaced by $\frac{\mu+1}{1-\frac{K}{v-c}}>1$, with the expressions for the cutoffs adjusted accordingly.

Thus, we have established the following result.

Proposition 9. *Suppose each seller starts with a measure one of captive buyers. Suppose the platform can charge a two-part tariff: a transaction fee f and a fixed fee F, subject to $F\geq -K$. The platform finds it optimal to induce both sellers to join. If $0\leq K<v-c$, then $F^*=-K$, $f^*<v-c+b$ and the optimal level of discovery is given by*

$$x^*=\begin{cases}1 & if \quad 0<\theta\leq\theta_1(\mu,K)\\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}\left(1-\frac{K}{v-c}\right)}}{2\theta-1} & if \quad \theta_1(\mu,K)\leq\theta\leq\theta_2(\mu,K)\\ 0 & if \quad \theta\geq\theta_2(\mu,K)\end{cases}$$

where

$$\theta_2(\mu,K)=\frac{\mu+1}{\mu+2-\frac{K}{v-c}}\in\left[\frac{1}{2},1\right]$$

and $\theta_1(\mu,K)\in\left(\frac{1}{2},\theta_2(\mu,K)\right)$ is the unique solution to

$$\frac{\theta}{(1-\theta)^3}=\frac{4(\mu+1)}{1-\frac{K}{v-c}}$$

The optimal level of discoverability x^* is decreasing in θ, and increasing in μ and K.

It is easily verified that setting $K = 0$ leads to the results in Proposition 1. The platform wants to offer a subsidy, but if it cannot, it optimally chooses no fixed fee. This means our baseline results still apply even if the platform can use two-part tariffs, provided moral hazard (or some other constraint) prevents the platform offering sellers fixed subsidies.

The reason the platform always chooses a subsidy here, if it can, reflects that the platform wants to push the final price charged by the two sellers up to the monopoly price v, thereby maximizing joint profit. The platform then extracts this profit subject to leaving each seller only with their outside option. Since in this Bertrand setting, the only way to achieve the monopoly price is to charge a transaction fee equal to the monopoly price, this leaves sellers with zero profit, which given their positive outside option, implies sellers must receive a subsidy to keep them willing to participate.[4]

The larger K, i.e. the more the platform can subsidize seller participation via a negative fixed fee, the higher the optimal level of discoverability (until $K \geq v - c$, at which point full discoverability is optimal). This makes sense: fixed subsidies are a way to compensate sellers for the individual downside of discoverability, and maximum discoverability is better from a joint profit perspective. Another way to understand the result is in terms of the usual tradeoff in setting discoverability: a higher fixed subsidy allows the platform to charge a higher transaction fee, which increases its profits from expanding the number of transactions, and shifts the tradeoff towards a higher level of discoverability.

A.9 Proof of Corollary 1

Recall the optimal level of discoverability in the baseline is

$$x_b^* = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \theta_1(\mu) \\ \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1} & \text{if} \quad \theta_1(\mu) \leq \theta \leq \frac{\mu+1}{\mu+2} \\ 0 & \text{if} \quad \theta \geq \frac{\mu+1}{\mu+2} \end{cases},$$

with $\theta_1(\mu) \in \left(\frac{1}{2}, \frac{\mu+1}{\mu+2}\right)$ the unique solution in θ to

$$\frac{\theta}{(1-\theta)^3} = 4(\mu+1).$$

[4] In more general models with imperfect competition, the platform may be able to induce the monopoly price while still leaving sellers with positive profits. Then whether the fixed fee is positive or negative depends on how these profits compare to the outside option, which is positive here. This is in contrast to traditional vertical relationship models, where it is never necessary to subsidize via the fixed fee given the outside option is typically assumed to give zero profits.

Consider first the case $\mu \geq 1$, so the optimal level of discoverability with differential fees is

$$x_{df}^* = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \theta_0(\mu) \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \theta_0(\mu) \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \text{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases},$$

where $\theta_0(\mu)$ is the unique solution to

$$\frac{\theta}{(1-\theta)^3} = 2(\mu+1),$$

so

$$\theta_0(\mu) < \theta_1(\mu)$$

Note that

$$\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

is equivalent to

$$\frac{1-\left(2-\sqrt{2}\right)\theta}{1-\theta}\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}} > 1.$$

The LHS is increasing in θ. Furthermore, it is easily verified that the inequality holds for $\theta = \frac{\mu+1}{\mu+2} < \frac{2(\mu+1)}{2(\mu+1)+1}$ and does not hold when $\theta = \theta_1(\mu)$. Thus, there exists $\theta_3 \in \left[\theta_1(\mu), \frac{\mu+1}{\mu+2}\right]$, such that the inequality holds for $\theta > \theta_3$ and does not hold for $\theta \leq \theta_3$. This implies the result for this case.

Now consider the case $\mu \leq 1$, so the optimal level of discoverability with differential fees is

$$x_{df}^* = \begin{cases} 1 & \text{if} \quad 0 < \theta \leq \frac{1}{2} \\ \frac{\mu}{(\mu+1)\theta} & \text{if} \quad \frac{1}{2} \leq \theta \leq \frac{2}{3+\mu} \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \frac{2}{3+\mu} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1} \\ 0 & \text{if} \quad \theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} \end{cases},$$

Note that

$$\frac{\mu}{(\mu+1)\theta} > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

is equivalent to

$$\frac{\theta(1-\mu)+\mu}{\theta\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}} < (\mu+1),$$

and the LHS of the last inequality is decreasing in θ. Furthermore, we still have

$$\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

iff

$$\frac{1-\left(2-\sqrt{2}\right)\theta}{1-\theta}\sqrt{\frac{\theta}{2\left(\mu+1\right)\left(1-\theta\right)}} > 1$$

and the LHS of the last inequality is increasing in θ. And

$$\frac{\mu}{\left(\mu+1\right)\theta} = \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$$

when $\theta = \frac{2}{3+\mu}$. Define

$$f\left(\theta\right) = \begin{cases} \frac{\mu}{(\mu+1)\theta} & \text{if} \quad \theta \leq \frac{2}{3+\mu} \\ \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} & \text{if} \quad \theta \geq \frac{2}{3+\mu} \end{cases}.$$

We have

$$f\left(\theta\right) < \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1} = 1$$

when $\theta = \theta_1\left(\mu\right)$ and

$$f\left(\theta\right) > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$$

when $\theta = \frac{\mu+1}{\mu+2}$.

So we can conclude there exists $\theta_3 \in \left[\theta_1\left(\mu\right), \frac{\mu+1}{\mu+2}\right]$ such that $f\left(\theta\right) > \frac{1-\sqrt{\frac{\theta}{(\mu+1)(1-\theta)}}}{2\theta-1}$ iff $\theta > \theta_3$, which implies the result for this case as well.

A.10 Proof with n sellers and less favorable beliefs

In the setting with n sellers, we can consider the role of different "beliefs" to see whether they matter given there are positive network effects: the willingness of a seller to join increases in the number of other sellers since their expected payoff of joining is increasing in the number of other sellers.

We define m-beliefs with $m \geq 2$ as follows. Whenever $\theta \leq 1-\frac{1}{n}$, each seller believes all other sellers will join if $f \leq \max\left\{b, b+\left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c)\right\}$ and believes no other seller will join otherwise. When $\theta > 1-\frac{1}{n}$, there is a unique equilibrium for any fee f so there is no role for beliefs for this range of θ.

Focusing on the case $\theta \leq 1 - \frac{1}{n}$ where the beliefs are defined, these beliefs imply that as long as $f \leq b$, the equilibrium where all sellers join will be selected. That's a reasonable way to select equilibrium given that if $f \leq b$, sellers will still want to join even if they think no other sellers will join. But m-beliefs restrict how much higher than this the platform can set f before sellers will coordinate on the equilibrium where no sellers join because they expect no other sellers to join (this equilibrium always exists when $f > b$).

To show the resulting equilibrium and ensure these beliefs always select a valid equilibrium, let $\theta_k = 1 - \frac{1}{k}$. Suppose first $\theta_n < \theta \leq 1$. This implies $b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c) < b$. Thus, if $f > b$, then no seller joins. If $b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c) < f \leq b$, then the only equilibrium is for exactly one seller will join. And if $f \leq b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c) \leq b$, then the only equilibrium is for all n sellers to join. As noted, for this range of θ, beliefs play no role.

Next suppose $\theta_m < \theta \leq \theta_n$. This means we have $b + \left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c) < b < b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c)$. This implies as long as $f \leq b$, the only equilibrium is all sellers join, because even if they believe no other seller joins they will want to join. This is consistent with m-beliefs. If $b < f \leq b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c)$, then there are only two possible equilibria: no seller joins (because each seller believes no other sellers will join) or all sellers join (because each seller believes enough other sellers will join). The stated m-beliefs select the equilibrium where no sellers join. And if $f > b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c) > b$, then the only equilibrium is no sellers join, consistent with m-beliefs.

Finally, suppose $\theta \leq \theta_m$. Then $b + \left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c) > b$, so if $f \leq b$, then the only equilibrium is for all sellers to join, while if $b < f \leq b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c)$, there are only two possible equilibria: no seller joins (because each seller believes no other sellers will join) or all sellers join (because each seller believes enough other sellers will join). Then the stated m-beliefs select the equilibrium where all sellers join if $f \leq b + \left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c)$ and select the equilibrium where no sellers join if $b + \left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c) < f \leq b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c)$. Finally, as before, if $f > b + \left(\frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}\right)(v-c) > b$, then the only equilibrium is no sellers join, consistent with m-beliefs.

With m-beliefs, the platform's profit-maximizing fee is

$$f_m(\theta) = \begin{cases} b + \frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}(v-c) & if \quad 0 \leq \theta \leq \theta_m \\ b & if \quad \theta_m < \theta \leq \theta_n \\ b + \frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}(v-c) & if \quad \theta_n < \theta \leq 1 \end{cases}.$$

Indeed, at this fee, all sellers participate in all cases, so there is no reason to charge anything less. Furthermore, when $\theta \leq \theta_n$, charging more than $\max\left\{b, b + \left(\frac{x(m(1-\theta)-1)}{x(m(1-\theta)-1)+1}\right)(v-c)\right\}$ results in no sellers participating and therefore zero sales. And when $\theta_n < \theta \leq 1$, charging f such that $b + \frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}(v-c) < f \leq b$ results in exactly one seller joining, so platform profits of at most λb, which is clearly dominated by charging $f = b + \frac{x(n(1-\theta)-1)}{x(n(1-\theta)-1)+1}(v-c)$, getting all sellers to join and setting x very close to zero, which yields close to $n\lambda b$ for the platform.

The platform's maximum profit is therefore

$$\Pi = f_m(\theta)\, n\lambda \left(1 + x(n-1)(1-\theta)\right).$$

When $0 \leq \theta \leq \theta_n$, this profit is easily seen to be increasing in x, so the platform will prefer full discoverability, i.e. $x^* = 1$, regardless of beliefs. And when $\theta > \theta_n$, the profit-maximizing fee $f_m(\theta)$ is also unaffected by beliefs (it does not depend on m), and is equal to the profit-maximizing fee for the case with favorable beliefs, resulting in the same x^* as in the main analysis with n sellers.

Thus, different beliefs do not change the platform's optimal choice of x^*. Note this is also consistent with the result we found when looking at the effect of beliefs with just two sellers. Note, however, that less favorable beliefs (lower m) do lower the platform's profits, as expected. Lower m lowers θ_m, so the range where the platform can set $f > b$ shrinks, and moreover even where $0 < \theta \leq \theta_m$, the fee $f(\theta)$ is increasing in m, so would also decrease.

A.11 Representative buyer with n sellers

Define utility as

$$U = v\sum_{i=1}^{n} q_i - (1-\theta)\sum_{i=1}^{n} q_i^2 - \frac{\theta}{2}\left(\sum_{i=1}^{n} q_i\right)^2$$

so

$$q_i = \frac{v-p_i}{2(1-\theta)} - \frac{\theta}{2(1-\theta)}\left(\frac{v - \frac{\sum_{i=1}^{n} p_i}{n}}{\theta + \frac{2(1-\theta)}{n}}\right).$$

Note can rewrite demand as

$$q_i = \frac{v-p_i}{2(1-\theta)+n\theta} - \frac{\theta\left(p_i - \frac{\sum_{i=1}^{n} p_i}{n}\right)}{2(1-\theta)\left(\theta + \frac{2(1-\theta)}{n}\right)}.$$

In case of a single seller selling to its captives, it would face

$$q_i = \frac{v - p_i}{2 - \theta},$$

which is the same as in the model with two sellers. A seller's profit in this case is

$$\pi_i = (p_i - c)\left(\frac{1}{2-\theta}\right)(v - p_i).$$

This is maximized by setting the monopoly price $p_i = \frac{v+c}{2}$, leading to a profit of

$$\pi_i = \left(\frac{1}{2-\theta}\right)\left(\frac{v-c}{2}\right)^2.$$

Consider pricing when $m - 1$ other sellers are expected to join the platform, so seller i sets p_i to maximize

$$\pi_i = (p_i + b - c - f)\left((1-x)\left(\frac{1}{2-\theta}\right)(v - p_i) + mx\left(\frac{v - p_i}{2(1-\theta) + m\theta} - \frac{\theta\left(p_i - \frac{\sum_{i=1}^{m} p_i}{m}\right)}{2(1-\theta)\left(\theta + \frac{2(1-\theta)}{m}\right)}\right)\right).$$

To understand this, note that a given seller brings its captive buyers to the platform to begin with. A fraction $1 - x$ of these buyers remain captive to the seller that brought them, so we get the demand faced when the seller is the only seller buyers can buy from. The platform then exposes the remaining fraction x to all other $m - 1$ sellers, which means all m sellers compete for this fraction x of buyers. At the same time, a fraction x of the buyers brought by every other seller is now exposed to all sellers, including the one we started with.

The FOC at the symmetric equilibrium is

$$\begin{aligned}
& (1-x)\left(\frac{1}{2-\theta}\right)(v - p_i) + mx\left(\frac{v - p_i}{2(1-\theta) + m\theta}\right) \\
& - (p_i + b - c - f)\left((1-x)\left(\frac{1}{2-\theta}\right) + \frac{mx}{2(1-\theta) + m\theta} + \frac{mx\theta\left(1 - \frac{1}{m}\right)}{2(1-\theta)\left(\theta + \frac{2(1-\theta)}{m}\right)}\right) \\
= \; & 0.
\end{aligned}$$

This implies

$$p^* = \frac{(v - (c + f - b))\left(\frac{(1-x)(2(1-\theta)+m\theta)}{2-\theta} + mx\right)}{\frac{2(1-x)(2(1-\theta)+m\theta)}{2-\theta} + 2mx + \frac{mx\theta(m-1)}{2(1-\theta)}} + (c + f - b).$$

A seller's profit in equilibrium is then

$$\pi^* = (p^* + b - c - f)(v - p^*)\left(\frac{(1-x)}{2-\theta} + \frac{mx}{2(1-\theta)+m\theta}\right).$$

Let $b = \mu(v - c)$. So a seller will join if $f \leq f^c$, assuming the other $m - 1$ sellers join. This is determined by

$$f^c = (1 + \mu - y(x,\theta))(v - c)$$

where

$$y(x,\theta) = \frac{\left(\frac{(1-x)(2(1-\theta)+m\theta)}{2-\theta} + mx + \frac{mx\theta(m-1)}{4(1-\theta)}\right)}{\left(\frac{(1-x)}{2-\theta} + \frac{mx}{(2(1-\theta)+m\theta)}\right)\sqrt{(2(1-\theta)+m\theta)\left(\begin{array}{c}(1-x)(2(1-\theta)+m\theta)\\ +mx(2-\theta) + \frac{mx\theta(2-\theta)(m-1)}{2(1-\theta)}\end{array}\right)}}.$$

The platform's profit provided $f \leq f^c$ is

$$\Pi(x,f) = fm\left((1-x)\left(\frac{1}{2-\theta}\right) + mx\left(\frac{1}{2(1-\theta)+m\theta}\right)\right)(v - p^*).$$

The platform's problem is to set f and x to maximize $\Pi(x,f)$ subject to

$$f \leq (1 + \mu - y(x,\theta))(v - c).$$

Figure 5 shows the optimal x^* for the case where we set $m = n$ and $n = 10$, where θ is on the horizontal axis and μ is on the vertical axis. Relative to figure 4, the darker regions in figure 5 have shrunk, indicating a reduction in the optimal level of discoverability. We have also confirmed that as n increases, the darker regions continue to shrink, indicating more sellers lead to a lower optimal level of discoverability.

A.12 Proof with a fixed consideration set

Suppose instead of buyers seeing all sellers when the platform makes sellers discoverable to x buyers, these x buyers instead see a fixed number $j - 1$ of other randomly selected sellers. As in our baseline, θ of the time these other sellers will be identical substitutes with the buyers buying only once from the lowest cost seller and $1 - \theta$ of the time these other sellers will be independent and lead to j sales from these buyers for a given seller. Given sellers are symmetric, all sellers obtain the same expected profit from charging the monopoly

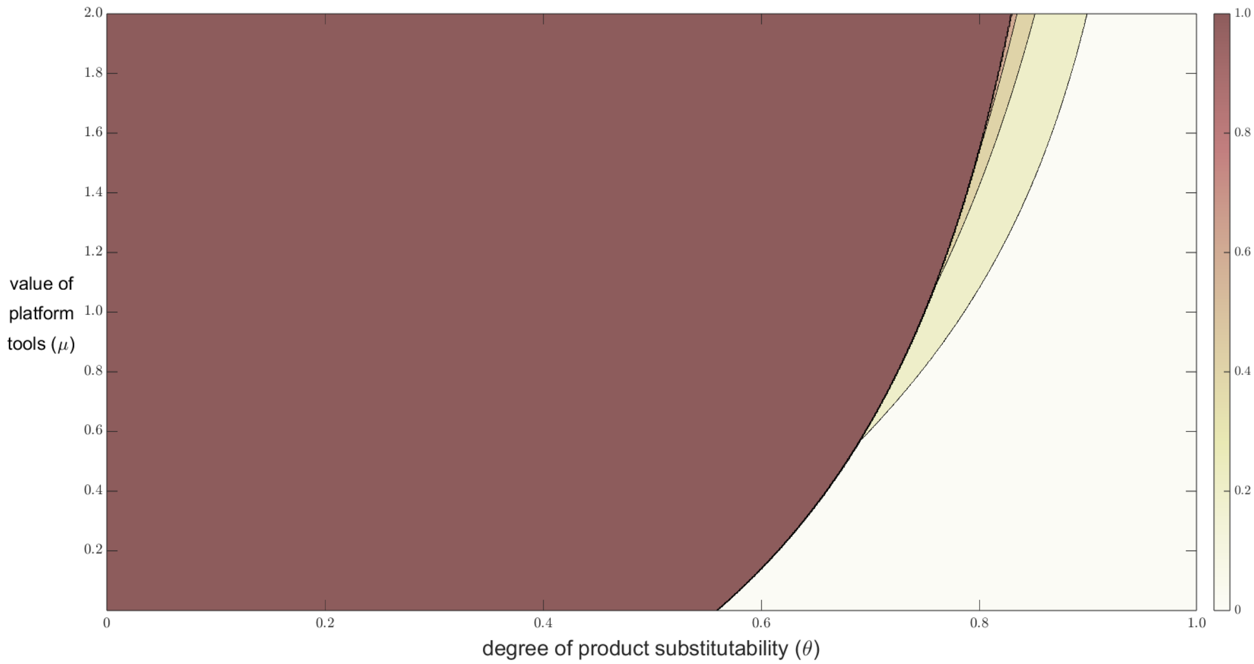

Figure 5: Heatmap of platform's optimal level of discoverability x^* (darker color indicates higher x^*)

price v to their captive buyers, which are expected to be

$$1 - x + xj(1 - \theta).$$

The analysis will be the same as if exactly j sellers are on the platform.

A.13 Working for differential fees case

Suppose first $f_0 = 0$. Then the platform is maximizing profit

$$\frac{(2-\theta)}{1-\theta}\left((v-c+b)\,x\,(1-2\theta)+b\right)$$

over x subject to

$$0 \leq \frac{(v-c)\,x\,(1-2\theta)+b\,(1+x\,(1-2\theta))}{x\,(1-\theta)} \leq v+b-c.$$

Clearly, the platform will set x such that $(v-c+b)\,x\,(1-2\theta)+b > 0$. So the only relevant constraint is

$$\frac{(v-c)\,x\,(1-2\theta)+b\,(1+x\,(1-2\theta))}{x\,(1-\theta)} \leq v+b-c,$$

which is equivalent to

$$\frac{\mu}{1+\mu} \leq x\theta$$

where $b = \mu(v-c)$. There are three cases:

1. If
$$\theta < \frac{\mu}{\mu+1},$$
then the constraint cannot be satisfied, so we can't have $f_0 = 0$.

2. If $\frac{\mu}{\mu+1} \leq \theta \leq \frac{1}{2}$, then $x^* = 1$ and the platform's maximum profits conditional on $f_0 = 0$ are
$$\begin{aligned} & \frac{(2-\theta)}{1-\theta}\left((v-c+b)(1-2\theta)+b\right) \\ = & \frac{(2-\theta)(1-2\theta)}{1-\theta}(v-c) + 2(2-\theta)b. \end{aligned}$$

3. If $\theta \geq \max\left\{\frac{\mu}{\mu+1}, \frac{1}{2}\right\}$, then $x^* = \frac{\mu}{(\mu+1)\theta}$ and the platform's maximum profits conditional on $f_0 = 0$ are
$$\begin{aligned} & \frac{(2-\theta)}{1-\theta}\left((v-c+b)\frac{\mu(1-2\theta)}{(\mu+1)\theta}+b\right) \\ = & b\frac{2-\theta}{\theta}. \end{aligned}$$

Now suppose $f_0 > 0$, so we must have

$$f_1 = (v-c)\frac{1-2\theta}{1-\theta} + b\frac{1+x(1-2\theta)}{x(1-\theta)} - f_0\frac{(1-x\theta)}{x(1-\theta)} = v+b-c,$$

which is equivalent to

$$f_0 = b - (v-c)\frac{x\theta}{1-x\theta} < v-c+b.$$

The platform's profits as a function of x are then

$$\begin{aligned} & -\left(b-(v-c)\frac{x\theta}{1-x\theta}\right)\frac{\theta(1-x)}{1-\theta} + \frac{(2-\theta)}{1-\theta}\left((v-c+b)x(1-2\theta)+b\right) \\ = & (v-c)\left(-\left(\mu-\frac{x\theta}{1-x\theta}\right)\frac{\theta(1-x)}{1-\theta} + \frac{(2-\theta)}{1-\theta}\left((1+\mu)x(1-2\theta)+\mu\right)\right) \\ = & (v-c)\left(1+2\mu+2(1+\mu)x(1-\theta) - \frac{1}{1-x\theta}\right). \end{aligned}$$

The platform maximizes these profits subject to $f_0 \geq 0$ (all other constraints are satisfied), which is equivalent to

$$x \leq \frac{\mu}{\theta\,(1+\mu)}.$$

The derivative of the last expression of platform profits above with respect to x is

$$(v-c)\left(2\,(1+\mu)\,(1-\theta)-\frac{\theta}{(1-x\theta)^2}\right),$$

so the second derivative is clearly negative, which means the SOC holds. The unconstrained optimal x is then

$$x^* = \frac{1-\sqrt{\frac{\theta}{2(1+\mu)(1-\theta)}}}{\theta}.$$

There are three cases.

1. If $\frac{\theta}{2(1+\mu)(1-\theta)} \geq 1$, which is equivalent to

   $$\theta \geq \frac{2\,(\mu+1)}{2\,(\mu+1)+1},$$

   then the optimal solution conditional on $f_0 > 0$ is $x^* = 0$, which implies $f_0 = b$ and the platform's profits are $2b$.

2. If

   $$0 \leq \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \leq \min\left\{1, \frac{\mu}{\theta\,(1+\mu)}\right\},$$

   which is equivalent to

   $$\frac{\theta}{(1-\theta)^3} \geq 2\,(\mu+1) \text{ and } \frac{2}{3+\mu} \leq \theta \leq \frac{2\,(\mu+1)}{2\,(\mu+1)+1},$$

   then the optimal solution conditional on $f_0 > 0$ is

   $$x^* = \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta},$$

   which implies $f_0 = b-(v-c)\,\frac{x^*\theta}{1-x^*\theta}$ and the platform's profits are

   $$(v-c)\left(1+2\mu+2\,(1+\mu)\,x^*\,(1-\theta)-\frac{1}{1-x^*\theta}\right).$$

3. If

$$\frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \geq \min\left\{1, \frac{\mu}{\theta(1+\mu)}\right\},$$

which is equivalent to

$$\frac{\theta}{(1-\theta)^3} \leq 2(\mu+1) \text{ or } \theta \leq \frac{2}{3+\mu},$$

then the optimal solution conditional on $f_0 > 0$ is

$$x^* = \min\left\{1, \frac{\mu}{\theta(1+\mu)}\right\},$$

which implies $f_0 = b - (v-c)\frac{x^*\theta}{1-x^*\theta}$ and the platform's profits are

$$(v-c)\left(1+2\mu+2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

The platform compares the best solution conditional on $f_0 = 0$ to the best solution conditional on $f_0 > 0$. We distinguish two cases: $\mu \leq 1$ and $\mu \geq 1$. Let $\theta_0(\mu)$ denote the unique solution to

$$\frac{\theta}{(1-\theta)^3} = 2(\mu+1).$$

Suppose first $\mu \leq 1$. Then we have

$$\frac{\mu}{\mu+1} \leq \theta_0(\mu) \leq \frac{1}{2} \leq \frac{2}{3+\mu} \leq \frac{2(\mu+1)}{2(\mu+1)+1}.$$

So:

- if $\theta \leq \frac{\mu}{1+\mu}$, then there is no solution with $f_0 = 0$, so the optimal solution is

$$\begin{aligned} x^* &= 1 \\ f_0 &= b-(v-c)\frac{\theta}{1-\theta} \\ f_1 &= v+b-c \end{aligned}$$

  and yields platform profits

$$(v-c)(2-\theta)\left(2(1+\mu) - \frac{1}{1-\theta}\right).$$

- if $\frac{\mu}{\mu+1} \leq \theta \leq \frac{1}{2}$, then the solution with $f_0 > 0$ has $x^* = \frac{\mu}{\theta(1+\mu)}$, which implies $f_0 = 0$. So this is weakly dominated by the solution conditional on $f_0 = 0$, which has

$$\begin{aligned} x^* &= 1 \\ f_1 &= \frac{(v-c)(1-2\theta)}{1-\theta} + 2b \end{aligned}$$

  and yields platform profits

$$(v-c)(2-\theta)\left(2(1+\mu) - \frac{1}{1-\theta}\right)$$

- If $\frac{1}{2} \leq \theta \leq \frac{2}{3+\mu}$, then the solution with $f_0 > 0$ has $x^* = \frac{\mu}{\theta(1+\mu)}$, which implies $f_0 = 0$. So this is weakly dominated by the solution conditional on $f_0 = 0$, which has

$$\begin{aligned} x^* &= \frac{\mu}{(\mu+1)\theta} \\ f_1 &= \frac{b(\mu+1)}{\mu} \end{aligned}$$

  and yields platform profits

$$(v-c)\mu\frac{2-\theta}{\theta}.$$

- if $\frac{2}{3+\mu} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution with $f_0 > 0$ has

$$\begin{aligned} x^* &= \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b-(v-c)\frac{x^*\theta}{1-x^*\theta} \\ f_1 &= v-c+b \end{aligned}$$

  and yields platform profits

$$(v-c)\left(1+2\mu+2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

  We know that $x^* = \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$ maximizes this last expression, so it must be higher than when it is evaluated at $x^* = \frac{\mu}{(\mu+1)\theta}$, where it is equal to $(v-c)\frac{(2-\theta)\mu}{\theta}$. The latter is the optimal platform profit that can be obtained conditional on $f_0 = 0$ (because $\theta \geq \frac{2}{3+\mu} > \frac{1}{2}$). So the optimal solution is the one above, with $f_0 > 0$.

- if $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution conditional on $f_0 > 0$ is

$$\begin{aligned} x^* &= 0 \\ f_0 &= b, \end{aligned}$$

with indeterminate f_1 and yielding platform profits

$$2b.$$

This dominates the solution with f_0, which yields $\frac{(2-\theta)b}{\theta}$, because $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} > \frac{2}{3}$.

Now suppose $\mu \geq 1$. Then we have

$$\frac{2}{3+\mu} \leq \frac{1}{2} \leq \theta_0(\mu) \leq \frac{\mu}{\mu+1} < \frac{2(\mu+1)}{2(\mu+1)+1}.$$

So:

- if $\theta \leq \theta_0(\mu)$, then there is no solution with $f_0 = 0$, so the optimal solution has

$$\begin{aligned} x^* &= 1 \\ f_0 &= b - (v-c)\frac{\theta}{1-\theta} \\ f_1 &= v + b - c \end{aligned}$$

yielding platform profits

$$(v-c)(2-\theta)\left(2(\mu+1) - \frac{1}{1-\theta}\right).$$

- if $\theta_0(\mu) \leq \theta \leq \frac{\mu}{\mu+1}$, then there is no solution with $f_0 = 0$, so the optimal solution has

$$\begin{aligned} x^* &= \frac{1 - \sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b - (v-c)\frac{x^*\theta}{1-x^*\theta} \\ f_1 &= v + b - c \end{aligned}$$

yielding platform profits

$$(v-c)\left(1 + 2\mu + 2(1+\mu)x^*(1-\theta) - \frac{1}{1-x^*\theta}\right).$$

- if $\frac{\mu}{\mu+1} \leq \theta \leq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the optimal solution with $f_0 > 0$ is

$$\begin{aligned} x^* &= \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta} \\ f_0 &= b-(v-c)\frac{x^*\theta}{1-x^*\theta} \\ f_1 &= v+b-c, \end{aligned}$$

yielding platform profits

$$(v-c)\left(1+2\mu+2(1+\mu)x^*(1-\theta)-\frac{1}{1-x^*\theta}\right).$$

We know that $x^* = \frac{1-\sqrt{\frac{\theta}{2(\mu+1)(1-\theta)}}}{\theta}$ maximizes this expression, so it must be higher than when it is evaluated at $x = \frac{\mu}{(\mu+1)\theta}$, where it is equal to $(v-c)\,\mu\frac{(2-\theta)}{\theta}$. The latter is the optimal profit that can be obtained conditional on $f_0 = 0$ (because $\theta \geq \frac{\mu}{\mu+1} \geq \frac{1}{2}$). So the optimal solution is the one above, with $f_0 > 0$.

- if $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1}$, then the solution conditional on $f_0 > 0$ is

$$\begin{aligned} x^* &= 0 \\ f_0 &= b, \end{aligned}$$

with indeterminate f_1 and yielding profits

$$2b.$$

This dominates the solution with f_0, which yields $\frac{(2-\theta)b}{\theta}$, because $\theta \geq \frac{2(\mu+1)}{2(\mu+1)+1} > \frac{2}{3}$.