Online Appendix for "Creating platforms by hosting rivals"

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This online appendix contains the formal details for various results noted in the main text, and the proofs behind some of the results and claims in the main text.

A Correlation in consumers' valuations across products

In the benchmark model, we assumed both types of consumers value A the same. We now explore what happens when the two types of consumers place different values on product A in the case without variable fees. Specifically, we assume B-types continue to value product A at u_A, but A-types value it at $u_A + \alpha$. We will consider both the case when α is positive (i.e. there is *negative* correlation between the values different types of consumers place on products A and B) and the case α is negative (i.e. there is *positive* correlation between the values different types of consumers place on products A and B).

A.1 Negative correlation

Suppose $0 < \alpha \leq \sigma$, so A-types are willing to pay α more for product A than are B-types. This captures the idea that there are some consumers who value A highly and do not need B (e.g. they may be serious body builders who go to the gym only to use the weightlifting equipment and have no time for cycling), while others are interested in both A and B, but value A relatively less (e.g. they go to the gym to for a variety of workouts). Comparing joint profits under hosting and without hosting, we obtain the following proposition.

Proposition 7 *If $\lambda_A \leq \frac{\sigma-\alpha}{u_A}$, then hosting is jointly preferred iff $\Delta > \frac{\lambda_A(u_A+\alpha-\sigma)+F}{2(1-\lambda_A)}$. If $\lambda_A > \frac{\sigma-\alpha}{u_A}$, then hosting is jointly preferred iff $\Delta > \frac{\sigma-\alpha}{2} + \frac{F}{2(1-\lambda_A)}$.*

Proof. Consider first what happens without hosting. Note if M charges a price of p_A, A-type consumers will buy A provided $p_A \leq u_A + \alpha - \sigma$. The choices of B-types is the same as our previous analysis with $\alpha = 0$. Recall M competes by selling both A and B_M to B-types. This allows it to increase its price to A-types to their maximum willingness to pay (now $u_A + \alpha - \sigma$), while still giving exactly the same surplus to B-types as before. Thus, without hosting there is a unique equilibrium outcome in which the prices are $p_A^* = u_A + \alpha - \sigma$, $p_B^* = \sigma - \alpha - \Delta$, $p_S^* = 0$. The A-type consumers always purchase A, and the B-type consumers all buy A and B_M from M. Profits are $\pi_M^* = u_A - \sigma + \lambda_A \alpha + \lambda_B (\sigma - \Delta)$ and $\pi_S^* = 0$. The proof follows the same steps as the proof of Proposition 1. As before, M cannot do better deviating. Note this remains true even if $p_B^* < 0$. If M sets $p_A = u_A - \sigma$ and sets a high p_B to induce multi-stop shopping, it will be worse off, since M would give up α on A-types and $\sigma - \Delta > 0$ on B-types. Moreover, the same alternative possibilities for equilibria can be ruled out using the same arguments as before, since M always does better setting the maximum price possible to sell to the A-types and adjusting p_B so as to compete with S. This logic, also rules out any equilibrium with a price $u_A - \sigma < p_A < u_A + \alpha - \sigma$.

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With hosting a similar tradeoff arises to before (i.e. whether to sell to all consumers or just B-types), except now the benefit of keeping A-type consumers is greater given they are willing to pay for product A. As before M has two options. Either it can set $p_A = u_A + \alpha - \sigma < u_A$ and sell A to all consumers, obtaining $\pi_M = u_A + \alpha - \sigma$, or set $p_A = u_A$ and sell A only to B-types, obtaining $\pi_M = \lambda_B u_A$. Then, we find (i) if $\lambda_A \leq \frac{\sigma - \alpha}{u_A}$, the selected equilibrium involves the prices $p_A^* = u_A$, $p_B^* = 0$, and $\widehat{p}_S^* = \Delta$, the A-type consumers do not purchase, while the B-type consumers all buy A and B_S through M, and profits are $\pi_M^* = \lambda_B u_A$ and $\pi_S^* = \lambda_B \Delta$; (ii) if $\lambda_A > \frac{\sigma - \alpha}{u_A}$, the selected equilibrium involves the prices $p_A^* = u_A + \alpha - \sigma$, $p_B^* = 0$, and $\widehat{p}_S^* = \Delta$, the A-type consumers always purchase A, and the B-type consumers all buy A and B_S through M, and profits are $\pi_M^* = u_A + \alpha - \sigma$ and $\pi_S^* = \lambda_B \Delta$.

The proposition follows by comparing the joint profit worked out above under hosting with joint profit under non-hosting, and taking into account the fixed cost of hosting F. ■

The tradeoff is similar to before, but there are some changes to note. The non-hosting profit extracted by M from B-types is not affected by α, since M can price discriminate: this means M's profit is just higher by the additional α obtained from A-types. By contrast, under hosting, the fact that $\alpha > 0$ means A-types are less of a constraint on the amount that M can extract from B-types since A-types are willing to pay more for A. This improves the profitability of hosting, unless M no longer wants to serve A-types under hosting, in which case M gives up more by hosting.

Consistent with this logic, a comparison of the regions under which hosting makes the firms jointly better off shows that hosting dominates for a larger range of Δ when M still sells to A-types (this occurs for large α), but dominates for a smaller range of Δ when M stops selling A-types (this occurs for small α). In the extreme case when $\alpha = \sigma$, the shopping cost is offset by the extra benefit that A-types get from product A, so A-types do not constrain at all the amount that M can extract from B-types even if it cannot price discriminate. Thus, apart from the fixed costs of hosting, hosting always dominates when $\alpha = \sigma$ as there is no other cost to hosting.

A.2 Positive correlation

Suppose instead that $\alpha < 0$, so B-type consumers are willing to pay more for both products than A-type consumers. Comparing joint profits under hosting and without hosting, we obtain the following proposition.

Proposition 8 *If $\lambda_A < -\frac{\alpha}{u_A - \sigma}$, then hosting is jointly preferred iff $\Delta > \frac{F}{2(1-\lambda_A)}$. If $-\frac{\alpha}{u_A - \sigma} \leq \lambda_A \leq \frac{\sigma - \alpha}{u_A}$, then hosting is jointly preferred iff $\Delta > \frac{\lambda_A (u_A - \sigma + \alpha) + F}{2(1-\lambda_A)} + \frac{\alpha}{2}$. If $\lambda_A > \frac{\sigma - \alpha}{u_A}$, then hosting is jointly preferred iff $\Delta > \frac{\sigma}{2} + \frac{F}{2(1-\lambda_A)}$.*

Proof. Consider first what happens without hosting. The equilibrium prices with non-hosting must satisfy $p_B^* \leq \sigma - \Delta$ in order for B-type consumers to prefer buying B_M to B_S, and $p_A^* + p_B^* \leq u_A - \Delta$ in order for B-type consumers to prefer buying A and B_M instead of just B_S, and $p_A^* \leq u_A + \alpha - \sigma$ if M sells to A-types or $p_A^* \leq u_A$ if M just sells to B-types. The new equilibria are characterized by:

NH-1 If $\lambda_A (\sigma - u_A) \leq \alpha < 0$ (or equivalently, $\lambda_A \geq -\frac{\alpha}{u_A - \sigma}$), then $p_A^* = u_A + \alpha - \sigma$, $p_B^* = \sigma - \Delta$, $p_S^* = 0$, with A-types still purchasing, and B-types buying the bundle from M, with profits being $\pi_M^* = u_A + \alpha - \lambda_A \sigma - \lambda_B \Delta$ and $\pi_S^* = 0$.

NH-2 If $\alpha < \lambda_A (\sigma - u_A) < 0$ (or equivalently, $\lambda_A < -\frac{\alpha}{u_A - \sigma}$), then M gives up on selling to A-types, $u_A - \sigma \leq p_A^* \leq u_A$ and $p_A^* + p_B^* = u_A - \Delta$, $p_S^* = 0$, with B-types buying the bundle from M, with profits being $\pi_M^* = \lambda_B (u_A - \Delta)$ and $\pi_S^* = 0$.

With hosting, the previous analysis with $\alpha \geq 0$ still holds, so the profit is defined in the proof of Proposition 7, in which there are two cases:

H-1 If $\lambda_A \leq \frac{\sigma-\alpha}{u_A}$, profits are $\pi_M^* = \lambda_B u_A$ and $\pi_S^* = \lambda_B \Delta$.

H-2 If $\lambda_A > \frac{\sigma-\alpha}{u_A}$, profits are $\pi_M^* = u_A + \alpha - \sigma$ and $\pi_S^* = \lambda_B \Delta$.

Note that $u_A + \alpha > \sigma$ (which is required for A-types to be willing to participate) implies the threshold $-\frac{\alpha}{u_A-\sigma}$ is smaller than the threshold $\frac{\sigma-\alpha}{u_A}$. Therefore, we have three cases when comparing the joint profits under hosting with non-hosting.

- If $\lambda_A < -\frac{\alpha}{u_A-\sigma}$, then NH-2 and H-1 apply, so we can compare $\lambda_B(u_A+\Delta) - F$ under hosting with $\lambda_B(u_A-\Delta)$ without hosting.
- If $-\frac{\alpha}{u_A-\sigma} \leq \lambda_A \leq \frac{\sigma-\alpha}{u_A}$, then NH-1 and H-1 apply, so we can compare $\lambda_B(u_A+\Delta) - F$ under hosting with $u_A + \alpha - \lambda_A\sigma - \lambda_B\Delta$ without hosting.
- If $\lambda_A > \frac{\sigma-\alpha}{u_A}$, then NH-1 and H-2 apply, so we can compare $u_A + \alpha - \sigma + \lambda_B\Delta - F$ under hosting with $u_A + \alpha - \lambda_A\sigma - \lambda_B\Delta$ without hosting.

The proposition follows by comparing the joint profit worked out above under hosting with joint profit under non-hosting. ■

The previous logic and tradeoff still apply. This suggests that $\alpha < 0$ tightens the constraint coming from A-types in the hosting equilibrium, thus making hosting less profitable. On the other hand, this also means that M loses less when it stops selling to A-types, which tends to make hosting more profitable. Finally, there is a novel effect when $\alpha < 0$: under non-hosting p_B is now constrained by competition in B (previously this constraint was not binding so M could adjust p_A and p_B to extract the maximum surplus from B-types). This limits M's ability to price discriminate, which previously was the key benefit provided by non-hosting. If α is sufficiently negative, then M no longer serves A-types under non-hosting, so in this case, if $F = 0$, then hosting always dominates. If M keeps selling to A-types under non-hosting, M's limited ability to benefit from price discrimination shifts the tradeoff in favor of hosting.

B Horizontal differentiation with respect to product B

Our results do not depend crucially on the assumption that B-type consumers are all the same. Consider the variation from our baseline model in which B-type consumers have heterogeneous tastes over products B_M and B_S. Specifically, suppose B-type consumers value B_M and B_S at u_B and $u_B + \Delta$ respectively, less their individual mismatch cost. Their mismatch cost is tx if purchasing B_M and $t(1-x)$ if purchasing B_S for a consumer located at x, where consumers have x drawn from $U[0,1]$. Thus, we model heterogeneous tastes using the standard Hotelling model of horizontal product differentiation. Other than this, we retain the assumptions of our baseline specification, and add a condition on the mismatch parameter t so that the market for B is always covered (t is not too high) and a condition on t so that both firms obtain positive markets shares in equilibrium both with and without hosting (t is not too low). Then we obtain the following proposition.

Proposition 9 *Suppose there is horizontal differentiation for product B, with the mismatch parameter t satisfying* $\max\left(\frac{\sigma-\Delta}{3}, \frac{\Delta}{3}\right) < t < \frac{2u_B}{3} + \min\left\{\frac{\sigma-\Delta}{9}, \frac{\Delta}{3}\right\}$. *When $\lambda_A \leq \frac{\sigma}{u_A}$, hosting is jointly preferred iff $\Delta > \frac{\sigma}{2} + \frac{9t(\lambda_A u_A - \sigma + F)}{2\sigma(1-\lambda_A)}$. When $\lambda_A > \frac{\sigma}{u_A}$, hosting is jointly preferred iff $\Delta > \frac{\sigma}{2} + \frac{9tF}{2\sigma(1-\lambda_A)}$.*

Proof. First consider the case without hosting. Note that $p_A \leq u_A$ otherwise M never sells A. We can also rule out M setting p_A such that $u_A - \sigma < p_A \leq u_A$, so A-types do not buy A. Suppose there is an equilibrium with this property. In this case B-types would not get a positive surplus from just buying A from M. Therefore, they either buy A and B_M from M or just B_S from S. It is straightforward to check that M will always prefer to set $p'_A = u_A - \sigma$ so as to sell to the A-types, and adjust the price for p_B to sell the bundle A and B_M at the same joint price $p_A + p_B$ as in the proposed equilibrium, which it can always do by setting a higher price for p_B.

Given $p_A \leq u_A - \sigma$, we know A-types will purchase and B-types who prefer to buy B_S will choose to multi-stop shop rather than one-stop shop at S. In this case, M does best setting $p_A = u_A - \sigma$, and the two firms' respective profits are $\pi_M = p_A + \lambda_B p_B \left(\frac{1}{2} + \frac{p_S - p_B + \sigma - \Delta}{2t}\right)$ and $\pi_S = \lambda_B p_S \left(\frac{1}{2} - \frac{p_S - p_B + \sigma - \Delta}{2t}\right)$. The equilibrium involves $p_A^* = u_A - \sigma$, $p_B^* = t + \frac{\sigma - \Delta}{3}$, $p_S^* = t - \frac{\sigma - \Delta}{3}$, $\pi_M^* = u_A - \sigma + 2t\lambda_B \left(\frac{1}{2} + \frac{\sigma - \Delta}{6t}\right)^2$ and $\pi_S^* = 2t\lambda_B \left(\frac{1}{2} - \frac{\sigma - \Delta}{6t}\right)^2$. It is straightforward to check that our assumptions on t imply S can earn a non-negative profit at these prices, both firms get some share of the B market, and the market is covered, and moreover that there is no profitable deviation for either firm.

Now suppose S is hosted by M. For the standard reasons, if $\lambda_A > \frac{\sigma}{u_A}$, M will set $p_A = u_A - \sigma$ and sell A to everyone, while if $\lambda_A \leq \frac{\sigma}{u_A}$, M will set $p_A = u_A$ and sell only to B-types. In either case, the equilibrium involves $p_B^* = t - \frac{\Delta}{3}$ and $\widehat{p}_S^* = t + \frac{\Delta}{3}$. As a result, if M sets $p_A = u_A$, profit are $\pi_M = \lambda_B u_A + 2t\lambda_B \left(\frac{1}{2} - \frac{\Delta}{6t}\right)^2$ and $\pi_S^* = 2t\lambda_B \left(\frac{1}{2} + \frac{\Delta}{6t}\right)^2$, while if M sets $p_A = u_A - \sigma$, profits are $\pi_M^* = u_A - \sigma + 2t\lambda_B \left(\frac{1}{2} - \frac{\Delta}{6t}\right)^2$ and $\pi_S^* = 2t\lambda_B \left(\frac{1}{2} + \frac{\Delta}{6t}\right)^2$. Our assumption on t ensures the market is covered, both firms get some share of the B market, and there is no profitable deviation for each firm. Note checking that there is no profitable deviation also requires checking that S would never want to set $p_S < \widehat{p}_S^*$ to induce some multi-stop shopping or some buyers to one-stop shop at S. Doing so will not attract any consumers to multi-stop shop unless $p_S < \widehat{p}_S^* - \sigma$. Since all B-type consumers buy A, to the extent they get some surplus from buying A, getting consumers to one-stop shop at S instead of at M will also require $p_S < \widehat{p}_S^* - (u_A - p_A)$. In both cases, S could attract more additional consumers by lowering $\widehat{p}_S$ instead of p_S by the given amount. The fact it doesn't want to (i.e. that $\widehat{p}_S^*$ is the equilibrium level of p_S) implies it also cannot be better off lowering p_S below $\widehat{p}_S^*$.

The proposition follows by comparing the joint profit worked out above under hosting with joint profit under non-hosting, taking into account the fixed cost of hosting F. ■

Note that the right-hand side in the tradeoff is always increasing in λ_A and F, which is consistent with the logic of the baseline model, namely that hosting is less likely for high λ_A and high F. If $\lambda_A \leq \frac{\sigma}{u_A}$, the right-hand side in the tradeoff is also increasing in u_A and decreasing in σ, which is also consistent with the logic in the baseline model. On the other hand, if $\lambda_A > \frac{\sigma}{u_A}$, the right-hand side in the tradeoff may be increasing or decreasing in σ, whereas in the baseline model it was always increasing. Finally, note the right-hand side of the tradeoff can be increasing or decreasing in the degree of product differentiation t when $\lambda_A < \frac{\sigma}{u_A}$ but is always increasing in the degree of product differentiation when $\lambda_A > \frac{\sigma}{u_A}$.

C Elastic demand by A-types

In this section we extend our analysis to the case in which A-types have elastic demand. We show that, in contrast to our benchmark setting, hosting may be unilaterally profitable without the full exclusion of A-types.

Suppose λ_A consumers get $u_A + \delta - y$ from consuming A, where $u_A > \sigma$, $\delta > 0$, and y is distributed with the weakly concave smooth distribution $G\,[0, u_A + \delta]$. Note $\delta > 0$ ensures that some A-types value product A more than B-types, so that the introduction of elastic demand by A-types does not have to imply lower willingness to pay by A-types. As before, λ_B consumers get $u_A > \sigma$ from consuming A, $u_B > \sigma$ from consuming B_M and $u_S = u_B + \Delta$ from consuming B_S, where $\sigma > \Delta \geq 0$. We assume

$$\arg\max_{p_A} \left\{ p_A G\left(u_A + \delta - \sigma - p_A\right) \right\} \leq u_A, \tag{C.1}$$

so the unconstrained price to maximize revenue from A-types is no more than u_A. This is a reasonable and simple condition to rule out equilibria in which M only sells to A-types, both with hosting and without hosting. Note with linear G, it just requires $\delta \leq u_A + \sigma$.

For conciseness, we assume in this section that there is no fixed cost of hosting, $F = 0$.

C.1 Non-hosting

Without hosting, the only equilibrium is one in which A-types with $y \leq u_A + \delta - \sigma - p_A$ buy from A, and the remainder do not, while B-types buy A and B from M and S sets $p_S^* = 0$. In this equilibrium, M chooses

$$\begin{aligned} p_A^* &= \arg\max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B \min\{u_A - \Delta, p_A + \sigma - \Delta\}\} \\ p_B^* &= \min\{u_A - \Delta - p_A^*, \sigma - \Delta\}. \end{aligned}$$

The corresponding profit for M is

$$\pi_M^* = \lambda_A p_A^* G(u_A + \delta - \sigma - p_A^*) + \lambda_B \min\{u_A - \Delta, p_A^* + \sigma - \Delta\}.$$

Note the equilibrium implies two possible outcomes: $u_A - \sigma < p_A^* \leq u_A + \min\{0, \delta - \sigma\}$ so that $p_B^* = u_A - \Delta - p_A^*$, or $p_A^* \leq u_A - \sigma$ so that $p_B^* = \sigma - \Delta$.

To show this is an equilibrium, we need to check that M cannot profitably deviate (clearly, there is no profitable deviation for S). First, M cannot do better if it just gives up on selling B_M but continues to sell A to B-types. Indeed, profits in such a deviation would be

$$\max_{p_A \leq u_A - \sigma} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B p_A\} < \pi_M^*,$$

since $\sigma > \Delta$.

Second, M cannot do better giving up on selling to B-types and just setting the unrestricted p_A to maximize its revenue from selling to A-types only. Indeed, consider the revenue-maximizing deviation price $p_A' = \arg\max_{p_A} \{p_A G(u_A + \delta - \sigma - p_A)\}$. By assumption (C.1), we must have $p_A' \leq u_A$. Furthermore, we must also have $p_A' < u_A + \delta - \sigma$, otherwise demand from A-types would be zero. Thus, we must have $p_A' \leq u_A + \min\{0, \delta - \sigma\}$. But this means the deviation cannot be profitable since

$$\begin{aligned} \pi_M^* &= \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B \min\{u_A - \Delta, p_A + \sigma - \Delta\}\} \\ &> \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A)\}. \end{aligned}$$

Thus, there is no profitable deviation from the proposed equilibrium.

Next we show that there are no equilibria involving (i) $p_A > u_A + \min\{0, \delta - \sigma\}$ or (ii) M only selling to A-types.

Suppose there is an equilibrium with $p_A > u_A + \min\{0, \delta - \sigma\}$. If $p_A > u_A + \max\{0, \delta - \sigma\}$, then M makes no sales of A whatsoever, which cannot be an equilibrium (M could lower p_A until either some A-types or some B-types buy A). Thus, there are two remaining possibilities. If $u_A < p_A \leq u_A + \delta - \sigma$, then M makes no sales of A to B-types and therefore no sales of B_M either. Then M's profits are $\lambda_A p_A G(u_A + \delta - \sigma - p_A)$, and assumption (C.1) implies that they can be increased by setting $p_A' \leq u_A$. The second possibility is $u_A + \delta - \sigma < p_A \leq u_A$. In this case, no A-types purchase from M and B-types either purchase both A and B_M from M or purchase nothing from M (indeed, B-types never purchase A alone given $u_A + \delta - \sigma < p_A$, and they don't purchase B

alone in equilibrium either given S offers a superior version of the B product). If B-types do not purchase from M, then M can always do better lowering its price p_A and at least selling to A-types. Suppose then that in the proposed equilibrium M sells both A and B_M to B-types, so S is not selling anything, and in the equilibrium sets $p_S = 0$. We must therefore have

$$\begin{aligned} u_A + u_B - p_A - p_B - \sigma &\geq u_B - \sigma + \Delta > 0 \\ u_A &\geq p_A > u_A + \delta - \sigma \\ u_B &\geq p_B. \end{aligned}$$

Note that the first two sets of inequalities imply $p_B < \sigma - \Delta - \delta$ and $p_A \leq u_A - p_B - \Delta$. But now M can do better by decreasing p_A slightly below $u_A + \delta - \sigma$ to attract some A-types and increase p_B by an offsetting amount so as to not change the total utility offered to B-types (i.e. so as not to lose any B-types) provided B-types still want to buy B_M at this higher price. To see this is possible, note that the proposed equilibrium prices satisfy $u_A - p_B - \Delta \geq p_A > u_A + \delta - \sigma$. Thus, we know the required increase in p_B will not be greater than $(u_A - p_B - \Delta) - (u_A + \delta - \sigma) = -p_B - \Delta - \delta + \sigma$. The deviation p_B' will therefore be lower than $p_B - p_B - \Delta - \delta + \sigma = \sigma - \Delta - \delta < u_B$, so this profitable deviation is indeed possible, which rules out the proposed equilibrium.

Finally, suppose there is an equilibrium in which M only sells to A-types. From the previous paragraph, in which we ruled out any equilibrium with $p_A > u_A + \min\{0, \delta - \sigma\}$, we know that we must have $p_A \leq u_A + \min\{0, \delta - \sigma\}$. But then, the strategy of selling to both A-types and B-types leading to π_M^* is strictly better, so must represent a profitable deviation from any such proposed equilibrium.[1]

C.2 Hosting

Now consider the case in which S is hosted by M and there is no variable fee charged by M. As in the benchmark case, in equilibrium S must win sales of B on M and we have $p_B^* = 0$, and $\widehat{p}_S^* = p_S^* = \Delta$. Thus, M only sells A in equilibrium. Furthermore, (C.1) implies M does not want to set $p_A > u_A$ and only sell to A-types. There are then two possibilities in equilibrium: (i) M sells A to both A-types and B-types by setting $p_A \leq u_A + \min\{0, \delta - \sigma\}$, (ii) M sells to B-types only by setting $u_A + \delta - \sigma < p_A \leq u_A$. Note that case (ii) is only possible if $\delta < \sigma$.

Consider case (i) first. In this equilibrium, M sets

$$p_A^* = \arg \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B p_A\},$$

and B-types prefer to buy A and B_S at M. The resulting profit for M is

$$\pi_M = \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B p_A\}.$$

If $\delta \geq \sigma$, then M cannot profitably deviate so this is clearly an equilibrium. If on the other hand $\delta < \sigma$, then M can deviate by setting $u_A + \delta - \sigma < p_A \leq u_A$ and thereby give up on A-types altogether. The best deviation in this case is attained for $p_A = u_A$ and yields profits $\lambda_B u_A$. Thus, provided $\delta \geq \sigma$ or

$$\max_{p_A \leq u_A + \delta - \sigma} \{\lambda_A p_A G(u_A + \delta - \sigma - p_A) + \lambda_B p_A\} \geq \lambda_B u_A, \tag{C.2}$$

[1] Recall, the equilibrium profits π_M^* were obtained assuming $p_S = 0$. If instead, S sets $p_S > 0$ in a proposed equilibrium in which M only sells to A-types, this would make the deviation to sell to both types even more profitable.

then the equilibrium is as characterized in (i).

If instead $\delta < \sigma$ and

$$\max_{p_A \leq u_A + \delta - \sigma} \left\{ \lambda_A p_A G\left(u_A + \delta - \sigma - p_A\right) + \lambda_B p_A \right\} < \lambda_B u_A,$$

then the equilibrium is as characterized in (ii), i.e. M sets $p_A^* = u_A$ and obtains profits $\pi_M = \lambda_B u_A$.

C.3 Comparison of hosting with non-hosting

In the benchmark setting, the only case in which hosting was unilaterally profitable for M is when $\lambda_A \leq \frac{\sigma}{u_A}$, and in this case M gives up on selling to A-types. We want to show that when A-types have elastic demand, hosting can now be unilaterally profitable without giving up on A-types altogether. Recall, there are two possible outcomes under hosting, corresponding to cases (i) and (ii) in the hosting analysis. In case (ii), M gives up on selling to A-types, so if M wants to host in this case, it is for a similar reason to that in the benchmark analysis.

The more interesting setting is case (i). In this case, hosting is unilaterally profitable for M iff

$$\begin{aligned} & \pi_A^{(i)} = \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \left\{ \lambda_A p_A G\left(u_A + \delta - \sigma - p_A\right) + \lambda_B p_A \right\} \\ > \quad & \pi_A^{nh} = \max_{p_A \leq u_A + \min\{0, \delta - \sigma\}} \left\{ \lambda_A p_A G\left(u_A + \delta - \sigma - p_A\right) + \lambda_B \min\left\{u_A - \Delta, p_A + \sigma - \Delta\right\} \right\}. \end{aligned}$$

We distinguish three cases.

In the first case, the value p_A^M maximizing $p_A G\left(u_A + \delta - \sigma - p_A\right)$ exceeds $u_A - \Delta$, which is indeed possible if $\delta + \Delta > \sigma$. Then hosting dominates non-hosting for all values of λ_A. In this case M benefits from being able to extract more than $u_A - \Delta$ from B-types under hosting, which it can do by setting a higher price for A because shopping costs are now taken care of by S through the surplus obtained from B_S. Thus, it sacrifices some, but not all, demand from A-types.

The second case is when $p_A^M < u_A - \Delta$ and $\delta > \sigma$. Then for any λ_A, M sells to some A-types when hosting. Notice that the optimal prices under hosting and non-hosting are both non-increasing with λ_A and above $u_A - \sigma$ for the same interval of values of λ_A.[2] On this interval, the price under non-hosting is $\max\left\{u_A - \sigma, p_A^M\right\}$ while the price under hosting decreases with λ_A until it reaches u_A (recall that $p_A G\left(u_A + \delta - \sigma - p_A\right)$ is weakly concave). Hosting can only dominate if λ_A is small enough that hosting induces an optimal price $p_A^* > u_A - \Delta$. On this range, $\left(\pi_A^{(i)} - \pi_A^{nh}\right)/\lambda_A$ is decreasing (the slope is $\left(u_A - \Delta - p_A^*\right)/\lambda_A^2$) and is positive for small λ_A, so that hosting dominates non-hosting for λ_A below a positive threshold.

The last case is when $p_A^M < u_A - \Delta$ and $\delta < \sigma$. The analysis is the same as in the previous case except that when λ_A increases, it reaches a level at which the case (ii) prevails. Depending on parameters values, the range of values of λ_A where hosting prevails and M sells to A-types may or may not be empty (as confirmed by solving the case when G is linear, with details available from the authors upon request).

D Consumer surplus and welfare

We evaluate the effect of hosting on consumer surplus and welfare in the benchmark case without variable fees, which comes from a straightforward comparison of the equilibria defined in Propositions 1 and 2. The results are summarized in the following proposition.

[2] If $u_A - \sigma \leq p_A^M \leq u_A - \Delta$, this is the case for all values of λ_A.

Proposition 10 *Consider the baseline model. If* $\lambda_A \leq \frac{\sigma}{u_A}$*, hosting lowers consumer surplus, and it increases total welfare if and only if* $\Delta > \frac{\lambda_A(u_A-\sigma)+F}{1-\lambda_A}$*. If* $\lambda_A > \frac{\sigma}{u_A}$*, hosting raises consumer surplus, and it increases total welfare if and only if* $\Delta > \frac{F}{1-\lambda_A}$.

The only parameter region where consumers are better off with hosting is the region in which M would individually prefer not to host. The reason is that hosting constrains the ability of M to extract profit from product A because M can no longer price discriminate. Only when this constraint is sufficiently important can consumer surplus be higher. While hosting increases competition over product B, it may not increase overall competition for the benefit of consumers when both products are taken into account.

It is intuitive that hosting increases total welfare by eliminating the additional shopping cost for B-type consumers to get A and B_S. This gives B-types an additional utility of Δ compared to when they were buying A and B_M without hosting. Other than the fixed cost F, the only other downside of hosting occurs when M stops selling to A-types, which happens when Δ is not very high. In this case, welfare can be lower with hosting even in the absence of any fixed cost (i.e. $F = 0$).

Thus, it is possible that hosting is jointly profitable but leads to lower total welfare. This happens when $\lambda_A \leq \frac{\sigma}{u_A}$ and $\frac{\lambda_A}{2(1-\lambda_A)}(u_A-\sigma) < \Delta < \frac{\lambda_A}{1-\lambda_A}(u_A-\sigma)$. Conversely, it is possible that hosting is not jointly profitable but leads to higher total welfare: this happens when $\lambda_A > \frac{\sigma}{u_A}$ and $\Delta < \frac{\sigma}{2}$.

E Hosting as information

We start by considering the case without hosting. The coexistence of informed and uninformed consumers implies that for some parameter range there is no equilibrium in pure strategies. In that case, we determine the (unique) equilibrium in mixed strategies. The following proposition summarizes the outcome without hosting.

Proposition 11 *When a fraction* η *of* B*-type consumers are uninformed of* S*'s existence, the non-hosting equilibrium is determined as follows:*

- *For* $\eta \leq \frac{\sigma-\Delta}{u_B}$*, the unique equilibrium is in pure strategies and is identical to the benchmark case (see Proposition 1).*
- *For* $\eta > \frac{\sigma-\Delta}{u_B}$*, the unique equilibrium is in mixed strategies and involves* $p_A = u_A - \sigma$*,* p_B *drawn from the CDF*

$$G_B(p_B) = \frac{p_B - \eta u_B}{p_B - \sigma + \Delta}$$

with support $p_B \in [\eta u_B, u_B]$ *and a mass point at* $p_B = u_B$*,*

$$\Pr(p_B = u_B) = \frac{\eta u_B + \Delta - \sigma}{u_B + \Delta - \sigma} < 1,$$

p_S *drawn from the CDF*

$$G_S(p_S) = 1 - \frac{\eta}{1-\eta}\frac{u_B + \Delta - \sigma - p_S}{p_S + \sigma - \Delta}$$

with support $p_S \in [\eta u_B + \Delta - \sigma, u_B + \Delta - \sigma]$*. Expected equilibrium profits are*

$$\begin{aligned}\pi_M &= u_A - \sigma + \lambda_B \eta u_B \\ \pi_S &= \lambda_B(1-\eta)(\eta u_B + \Delta - \sigma).\end{aligned}$$

Proof. We first show there cannot be any pure-strategy equilibrium in which M sells B_M to uninformed B-types only. We then characterize the conditions under which there is a pure-strategy equilibrium in which M sells B_M to both informed and uninformed B-types. Finally, we characterize the mixed-strategy equilibrium, which turns out to exist if and only if the pure-strategy equilibrium does not exist.

To show there is no pure-strategy equilibrium in which M sells B_M to uninformed B-type consumers only, note this can only be an equilibrium if M extracts the entire surplus from uninformed B-types, which means we must have $p_A + p_B = u_A + u_B - \sigma$. Furthermore, M can extract the entire surplus from A-types by setting $p_A = u_A - \sigma$ and $p_B = u_B$. Given these prices, S's best response is to price just below $p_S = u_B + \Delta - \sigma$, which extracts almost the entire surplus from informed B-types. However, M could then deviate by slightly lowering p_B and attracting all informed B-type consumers as well, which results in a discrete increase in its profits. Thus, this cannot be part of a pure-strategy equilibrium.

Consider instead a possible pure-strategy equilibrium in which M sells to both informed and uninformed B-types. There are two cases to consider within this scenario: (a) M sets $p_A = u_A$ and only sells to B-types, and (b) M sets $p_A = u_A - \sigma$ and sells to both A-types and B-types. Case (a) is easily ruled out. Given that M sells to both informed and uninformed B-types, for this to be an equilibrium, we must have $p_B = -\Delta$ and $p_S = 0$, so M's profits are $\lambda_B (u_A - \Delta)$. But then M could deviate to $p_A = u_A - \sigma$ and $p_B = \sigma - \Delta$, which yields strictly higher profit $u_A - \sigma + \lambda_B (\sigma - \Delta)$.

Consider case (b) in which $p_A = u_A - \sigma$. We must have $p_S = 0$ and $p_B = \sigma - \Delta$. Thus, profits are $\pi_M = u_A - \sigma + \lambda_B (\sigma - \Delta)$ and $\pi_S = 0$. Clearly, S cannot profitably deviate. M has three possible deviations. The first one is to set $p_B = u_B$ in order to only attract the uninformed B-types, in which case it must keep $p_A = u_A - \sigma$. This deviation yields profits $u_A - \sigma + \lambda_B \eta u_B$. The second possible deviation is to set $p_A > u_A - \sigma$ and $p_A + p_B = u_A - \Delta$, which ensures that M still sells to all informed and uninformed B-types but no longer sells to A-types. This yields profits $\lambda_B (u_A - \Delta)$, which can never be a profitable deviation. The third possible deviation is to set $p_A > u_A - \sigma$ and $p_A + p_B = u_A + u_B - \sigma$, which ensures that M only sells to uninformed B-types. This yields profits $\lambda_B \eta (u_A + u_B - \sigma)$, which are lower than the profits obtained through the first deviation. Thus, the pure-strategy equilibrium under case (b) exists if and only if $u_A - \sigma + \lambda_B (\sigma - \Delta) \geq u_A - \sigma + \lambda_B \eta u_B$, which is equivalent to $\eta < \frac{\sigma - \Delta}{u_B}$. Thus, the only possible pure-strategy equilibrium has $p_A = u_A - \sigma$, $p_B = \sigma - \Delta$ and $p_S = 0$, and it exists if and only if $\eta \leq \frac{\sigma - \Delta}{u_B}$.

We next determine the mixed-strategy equilibria. Denote by G_B the CDF of M's price distribution for p_B and by G_S the CDF of S's price distribution for p_S. Again, there are two possibilities: (a) $p_A = u_A$, so M sells only to B-types, and (b) $p_A = u_A - \sigma$ so M sells to both A-types and some B-types.

As before we can rule out case (a) arising in equilibrium. To see this, suppose $p_A = u_A$, so we must have $p_B \leq u_B - \sigma$. In this case, for any p_B that M plays with positive probability, its profit is

$$\lambda_B (u_A + p_B) (\eta + (1 - \eta) \Pr (p_B < p_S - \Delta)) = \lambda_B (u_A + p_B) (\eta + (1 - \eta) (1 - G (p_B + \Delta))) .$$

However, by setting $p_A = u_A - \sigma$ and $\widetilde{p}_B = p_B + \sigma$, the profit achieved by M becomes

$$\begin{aligned} & u_A - \sigma + \lambda_B \widetilde{p}_B (\eta + (1 - \eta) (1 - G (\widetilde{p}_B - \sigma + \Delta))) \\ = \quad & u_A - \sigma + \lambda_B (p_B + \sigma) (\eta + (1 - \eta) (1 - G (p_B + \Delta))) , \end{aligned}$$

which is strictly higher. Thus, setting $p_A = u_A$ cannot be part of an equilibrium.

Turning next to case (b), given $p_A = u_A - \sigma$, we must have $p_B \leq u_B$. In particular, M can guarantee profits $u_A - \sigma + \lambda_B \eta u_B$ by setting $p_B = u_B$. This implies that M will never set p_B below ηu_B (even if it attracted both informed and uninformed consumers at this price, it would not do better than ηu_B in total profits from selling B_M). Thus, the support of $G_B(.)$ is $[\eta u_B, u_B]$. This implies that the support of $G_S(.)$ is $[\eta u_B + \Delta - \sigma, u_B + \Delta - \sigma]$.

We determine $G_S(.)$ by imposing that any price p_B in the support of G_B yields the same profit as setting $p_B = u_B$. This is equivalent to:

$$u_A - \sigma + p_B \lambda_B (\eta + (1-\eta)(1 - G_S(p_B + \Delta - \sigma))) = u_A - \sigma + \lambda_B \eta u_B.$$

Rearranging and with the change of variables $p_S \equiv p_B + \Delta - \sigma$, this is equivalent to

$$G_S(p_S) = 1 - \frac{\eta}{1-\eta} \frac{u_B + \Delta - \sigma - p_S}{p_S + \sigma - \Delta}.$$

Note $G_S(p_S)$ is increasing in p_S, and $G_S(u_B + \Delta - \sigma) = 1$ and $G_S(\eta u_B + \Delta - \sigma) = 0$, so $G_S(.)$ has no mass points.

Similarly, we determine $G_B(.)$ by imposing that any price p_S in the support of G_S yields the same profit as setting $p_S = \eta u_B + \Delta - \sigma$, thereby capturing all the informed customers. This is equivalent to

$$\lambda_B (1-\eta)(1 - G_B(p_S + \sigma - \Delta)) p_S = \lambda_B (1-\eta)(\eta u_B + \Delta - \sigma).$$

Rearranging and with the change of variables $p_B \equiv p_S - \Delta + \sigma$, this is equivalent to

$$G_B(p_B) = \frac{p_B - \eta u_B}{p_B - \sigma + \Delta}.$$

Note that $G_B(\eta u_B) = 0$. Furthermore, $G_B(p_B)$ is increasing in p_B if and only if $\eta > \frac{\sigma-\Delta}{u_B}$. If $\eta \leq \frac{\sigma-\Delta}{u_B}$, then $G_B(.)$ is weakly decreasing and therefore there is no mixed-strategy equilibrium. Note also that $\eta > \frac{\sigma-\Delta}{u_B}$ implies $G_B(u_B) < 1$, so there is a mass point at $p_B = u_B$,

$$\Pr(p_B = u_B) = \frac{\eta u_B + \Delta - \sigma}{u_B + \Delta - \sigma} < 1.$$

In this mixed-strategy equilibrium, expected profits are

$$\begin{aligned} \pi_M &= u_A - \sigma + \lambda_B \eta u_B \\ \pi_S &= \lambda_B (1-\eta)(\eta u_B + \Delta - \sigma). \end{aligned}$$

Finally, we need to check that M cannot profitably deviate by setting $p_A = u_A$ and thereby giving up selling to A-types. If it did, then M would have to set $p_B \leq u_B - \sigma$. When M sets p_B, it sells to informed B-types only if $p_S > p_B + \Delta$. Thus, M's deviation profits as a function of p_B are

$$\begin{aligned} & (u_A + p_B) \lambda_B (\eta + (1-\eta)(1 - G_S(p_B + \Delta))) \\ = \; & (u_A + p_B) \lambda_B \eta \frac{u_B}{p_B + \sigma}, \end{aligned}$$

which is decreasing in p_B. Thus, M's best deviation is to set $p_B = \eta u_B - \sigma$. The deviation profits are then $\lambda_B (u_A + \eta u_B - \sigma)$, which is clearly lower than the equilibrium profits $u_A - \sigma + \lambda_B \eta u_B$. Thus, the deviation is not profitable and the mixed-strategy equilibrium we have determined exists if and only if $\eta > \frac{\sigma-\Delta}{u_B}$. ■

Next consider the case with hosting. In principle, the fact that some consumers do not know about the specialist's existence before visiting M means that this scenario is somewhat different from the benchmark hosting case, where all consumers were informed of S's existence and presence on M even before going to M. However, it turns out that this difference does not affect the analysis (since all consumers are induced to shop at M in any equilibrium, they all end up informed of S's existence). Thus, the same hosting equilibrium as in the benchmark case prevails—both with and without variables fees. For convenience, the next proposition summarizes the joint profits under hosting for each case.

Proposition 12 *When a fraction η of B-type consumers are uninformed of S's existence, the hosting equilibrium and firm profits are the same as in the benchmark case:*

- *When M cannot monitor S's sales and charge variable fees, if $\lambda_A \leq \frac{\sigma}{u_A}$, then joint profits in equilibrium are $\lambda_B (u_A + \Delta) - F$, whereas if $\lambda_A > \frac{\sigma}{u_A}$, then joint equilibrium profits are $u_A - \sigma + \lambda_B \Delta - F$.*
- *When M can monitor S's sales and charge variable fees, if $\lambda_A \leq \frac{\sigma + \min\{0, u_B - 2\sigma\}}{u_A + \min\{0, u_B - 2\sigma\}}$, then joint profits are $\lambda_B (u_A + \Delta + \min\{\sigma, u_B - \sigma\}) - F$, whereas if $\lambda_A > \frac{\sigma + \min\{0, u_B - 2\sigma\}}{u_A + \min\{0, u_B - 2\sigma\}}$, then joint profits are $u_A - \sigma + \lambda_B (\sigma + \Delta) - F$.*

We can now compare the outcomes under hosting and non-hosting using the previous two propositions. The first proposition below focuses on the case when variable fees cannot be used.

Proposition 13 *Suppose a fraction η of B-type consumers are uninformed of S's existence under non-hosting, but they become informed about S under hosting if they visit M. When variable fees cannot be used:*

- *If $\eta \leq \frac{\sigma - \Delta}{u_B}$, the conditions for hosting to be jointly preferred are identical to those in Proposition 4.*
- *If $\eta > \frac{\sigma - \Delta}{u_B}$ and $\lambda_A \leq \frac{\sigma}{u_A}$, hosting is jointly preferred if and only if*

$$\Delta > \frac{F - (\sigma - \lambda_A u_A)}{\eta (1 - \lambda_A)} + \frac{(2 - \eta)(\eta u_B - \sigma)}{\eta} + \frac{\sigma}{\eta}.$$

- *If $\eta > \frac{\sigma - \Delta}{u_B}$ and $\lambda_A > \frac{\sigma}{u_A}$, hosting is jointly preferred if and only if*

$$\Delta > \frac{F}{\eta (1 - \lambda_A)} + \frac{(2 - \eta)(\eta u_B - \sigma)}{\eta} + \frac{\sigma}{\eta}.$$

The first case in Proposition 13 is identical to the benchmark analysis since, as explained above, the equilibrium analysis with hosting is unchanged and the equilibrium analysis without hosting is also unchanged when there are not many uninformed consumers. In the remaining two cases in Proposition 13, when the fraction of uninformed consumers is sufficiently high, the tradeoff unambiguously shifts towards non-hosting. To see this note that joint profit is the same under hosting, but the joint profits without hosting are higher in Proposition 11 when $\eta > \frac{\sigma - \Delta}{u_B}$ than they are in Proposition 1. When there are enough of them, the presence of uninformed consumers softens the competition for B-types without hosting, reflecting that M will sometimes exploit the uninformed B-types by setting a high price, and that S will best respond by also sometimes setting a high price p_S. In contrast, by promoting the specialist, hosting removes the friction that prevented S from reaching all consumers, thereby intensifying competition.

The effect of the factors (u_A, λ_A, F, σ and Δ) on the tradeoff between hosting and non-hosting in Proposition 13 is qualitatively the same as in Proposition 4, with two exceptions when $\eta > \frac{\sigma - \Delta}{u_B}$. First, when $\lambda_A > \frac{\sigma}{u_A}$, the tradeoff now shifts towards hosting when σ increases, reflecting that the shopping cost σ limits the surplus that can be extracted from uninformed B-types when competition is relaxed. Second, an increase in u_B now affects the tradeoff, shifting it towards non-hosting. This reflects that with higher u_B, the benefit of relaxing competition by focusing on exploiting uninformed B-types under non-hosting is higher since there is more surplus that can be extracted from such consumers.

For the case in which variable fees can be used by M under hosting, we make the comparison based on the joint profits under hosting (as specified in Proposition 5) with non-hosting (as given in Proposition 11). The comparison is summarized by Proposition 6. To show that the tradeoff unambiguously shifts in favor of non-hosting in Proposition 6 compared to in Proposition 5 in case $\eta > \frac{\sigma-\Delta}{u_B}$, note there are two cases.

Suppose $\lambda_A \leq \frac{\sigma+\min\{0,u_B-2\sigma\}}{u_A+\min\{0,u_B-2\sigma\}}$. If hosting is preferred in the case with η uninformed B-types, then

$$\begin{aligned}
\Delta &> \frac{F-(\sigma-\lambda_A u_A)}{\eta(1-\lambda_A)} + \frac{(2-\eta)(\eta u_B-\sigma)-\min\{0,u_B-2\sigma\}}{\eta} \\
&\iff \eta(1-\lambda_A)\Delta > F-(\sigma-\lambda_A u_A)+(1-\lambda_A)(2-\eta)(\eta u_B-\sigma)-(1-\lambda_A)\min\{0,u_B-2\sigma\} \\
&\iff (\eta-2)(1-\lambda_A)\Delta > F-(\sigma-\lambda_A u_A)-2\Delta(1-\lambda_A)+(1-\lambda_A)(2-\eta)(\eta u_B-\sigma)-(1-\lambda_A)\min\{0,u_B-2\sigma\} \\
&\iff 2\Delta(1-\lambda_A)-(F-(\sigma-\lambda_A u_A)-(1-\lambda_A)\min\{0,u_B-2\sigma\}) > (1-\lambda_A)(2-\eta)(\eta u_B-\sigma+\Delta) \\
&\iff \Delta - \frac{F-(\sigma-\lambda_A u_A)-(1-\lambda_A)\min\{0,u_B-2\sigma\}}{2(1-\lambda_A)} > \frac{(2-\eta)(\eta u_B-\sigma+\Delta)}{2} > 0,
\end{aligned}$$

where the first inequality comes from Proposition 6, and the last inequality holds because $\frac{\sigma-\Delta}{u_B} < \eta < 1$. Thus, we get that

$$\Delta > \frac{F-(\sigma-\lambda_A u_A)-(1-\lambda_A)\min\{0,u_B-2\sigma\}}{2(1-\lambda_A)},$$

which shows that hosting is then also preferred in the case without any uninformed B-types.

Suppose alternatively that $\lambda_A > \frac{\sigma+\min\{0,u_B-2\sigma\}}{u_A+\min\{0,u_B-2\sigma\}}$. If hosting is preferred in the case with η uninformed B-types, then following the same steps,

$$\begin{aligned}
\Delta &> \frac{F}{\eta(1-\lambda_A)} + \frac{(2-\eta)(\eta u_B-\sigma)}{\eta} \\
&\iff \Delta - \frac{F}{2(1-\lambda_A)} > \frac{(2-\eta)(\eta u_B-\sigma+\Delta)}{2} > 0.
\end{aligned}$$

Thus, we get that

$$\Delta > \frac{F}{2(1-\lambda_A)},$$

which shows that hosting is then also preferred in the case without any uninformed B-types.

Finally, we show how the comparative static results change in this case with uninformed B-types. The effect of the factors (u_A, λ_A, F, σ and Δ) on the tradeoff between hosting and non-hosting in Proposition 6 is qualitatively the same as in the corresponding case with full information (Proposition 5), with two exceptions: (i) when $\eta > \frac{\sigma-\Delta}{u_B}$, the tradeoff now always shifts towards hosting when σ increases, reflecting that the shopping cost σ limits the surplus that can be extracted from uninformed B-types when competition is relaxed (with full information, the tradeoff can shift towards non-hosting when σ increases), and (ii) when $\eta > \frac{\sigma-\Delta}{u_B}$ and either $u_B \geq 2\sigma$ or $\lambda_A > \frac{\sigma+\min\{0,u_B-2\sigma\}}{u_A+\min\{0,u_B-2\sigma\}}$, an increase in u_B shifts the tradeoff towards non-hosting as the benefit of relaxing competition by focusing on exploiting uninformed B-types under non-hosting is higher since there is more surplus that can be extracted from such consumers (with full information, u_B had no effect on the tradeoff in this parameter range).

F Multiple specialists and uncertainty

Suppose there are $n \geq 2$ specialists who offer $u_B + \Delta$ when visited outside M. When any of these specialists is hosted, we also assume there is uncertainty regarding the value offered to M's customers: for each hosted specialist, the value is $u_B + \Delta$ with probability θ and u_B with probability $1 - \theta$, where the realizations for different specialists are drawn independently and are the same for all consumers. This captures the general idea that there may be uncertainty over how the specialists will perform when hosted. Specifically, firms may be uncertain whether a given specialist's value added is specific to its location or carries over when it is hosted on M. This uncertainty is assumed to be resolved after the contract has been signed and the specialists have been hosted. Thus, if M decides to host one or multiple specialists, we assume M must commit to the fixed transfer and any variable fee τ before the uncertainty is resolved, while the firms set their prices (to consumers) after the uncertainty is resolved. Finally, we assume the fixed cost of hosting is F regardless of how many specialists are hosted. Note the case in which $\theta = 1$ captures the simple extension of the benchmark model to allow for multiple specialists (i.e. without uncertainty).

In the absence of hosting, allowing for multiple competing specialists does not change anything since in the benchmark case the specialist was already at a disadvantage when competing against M for sales of B (given $\Delta < \sigma$). Thus, all specialists price at zero and make zero profits, whereas M sets $p_A = u_A - \sigma$ and $p_B = \sigma - \Delta$, obtaining profits $u_A - \sigma + \lambda_B (\sigma - \Delta)$. This is the same outcome as in the benchmark case. With hosting, two differences arise: competition outside the platform drives prices to zero, and two hosted specialists offering the same value obtain zero profit. M must then decide how many specialists to host, and compare the resulting joint profits with the no-hosting outcome in deciding whether to host them.

The next two propositions characterize the hosting outcome and provide the conditions for hosting to be jointly preferred to non-hosting, first for the case when M cannot use variable fees under hosting (Proposition 14) and then for the case when M can use variable fees (Proposition 15). The proofs are provided at the end of the section.

Proposition 14 *Suppose there are $n \geq 2$ specialists who offer $u_B + \Delta$ when visited outside M, but when hosted offer $u_B + \Delta$ with probability θ and u_B with probability $1 - \theta$. When variable fees cannot be used:*

- *If $\sigma \leq \lambda_A u_A$, then M prefers to host $k^* = \arg\max_{k \in \{1,2,\ldots,n\}} \left\{ k (1-\theta)^{k-1} \right\}$ specialists, and hosting is jointly preferred if and only if*

$$\Delta > \frac{\sigma (1 - \lambda_A) + F}{(1 - \lambda_A) \left(1 + \theta k^* (1-\theta)^{k^*-1}\right)}.$$

- *If $\lambda_A u_A < \sigma < \lambda_A u_A + \lambda_B \Delta$, then M prefers to host*

$$k^* = \arg \max_{k \in \{1,2,\ldots,n\}} \left\{ (1-\theta)^k (\lambda_A u_A - \sigma) + k\theta (1-\theta)^{k-1} (\lambda_A u_A - \sigma + \lambda_B \Delta) \right\}$$

specialists and hosting is jointly preferred if and only if

$$\Delta > \frac{\sigma (1 - \lambda_A) - \left(1 - (1-\theta)^{k^*} - k^* \theta (1-\theta)^{k^*-1}\right) (\sigma - \lambda_A u_A) + F}{(1 - \lambda_A) \left(1 + k^* \theta (1-\theta)^{k^*-1}\right)}.$$

- *If $\sigma \geq \lambda_A u_A + \lambda_B \Delta$, then M prefers to host all n specialists and hosting is jointly preferred if and only if*

$$\Delta > \frac{\lambda_A (u_A - \sigma) + F}{(1-\lambda_A)(1-(1-\theta)^n)}.$$

By ignoring integer constraints and focusing on the case in which M hosts specialists, we can gain more insight into the optimal number of specialists to host. This is given by

$$k^* = \begin{cases} -\frac{1}{\ln(1-\theta)} & \text{if} \quad \sigma \leq \lambda_A u_A \\ -\frac{1}{\ln(1-\theta)} + \frac{(1-\theta)(\sigma - \lambda_A u_A)}{\theta(\lambda_A u_A + \lambda_B \Delta - \sigma)} & \text{if} \quad \lambda_A u_A < \sigma < \lambda_A u_A + \lambda_B \Delta \\ n & \text{if} \quad \sigma \geq \lambda_A u_A + \lambda_B \Delta \end{cases},$$

assuming k^* belongs to $[1, n]$ in each case. The optimal number of specialists to host is (weakly) increasing in σ, and is decreasing in θ, Δ and u_A.

To understand the result, note first that if $\sigma \geq \lambda_A u_A + \lambda_B \Delta$, then shopping costs are so high that, regardless of how many hosted specialists turn out to be of high value, M does best to sell A only to B-type consumers, who have their shopping cost covered by buying the B product. In this case, all specialists make zero profits, while M's profits are $\lambda_B u_A$, except in the case when no hosted specialist turns out to be of high value, when M's profits are $\lambda_B (u_A - \Delta)$. Thus, joint profits are higher when at least one of the specialists hosted on M turns out to offer the added value Δ, because the hosted specialists free the platform from outside competitive pressure on good A that arises when the value of good B is larger outside than inside. Since the probability of at least one specialist turning out to offer high value is increasing in k, M will host all available specialists.

On the other hand, if $\sigma \leq \lambda_A u_A$, then shopping costs are so low that, regardless of how many hosted specialists turn out to be of high value, M does best to sell A to all consumers by setting the price $p_A = u_A - \sigma$. In this case, joint profits are the same (equal to $u_A - \sigma$) across the different realizations, unless exactly one hosted specialist turns out to offer high value—then the two firms can extract the additional value Δ from B-types since it will not be competed away. Thus, M chooses k to maximize the probability $k\theta(1-\theta)^{k-1}$ of exactly one specialist offering high value. The optimal k is therefore decreasing in the probability θ that any individual specialist turns out to be of high value.

For intermediate levels of the shopping costs (i.e. $\lambda_A u_A < \sigma < \lambda_A u_A + \lambda_B \Delta$), M does best selling A only to B-types when two or more specialists turn out to be of high value. Otherwise, M does best selling A to all consumers. Thus, the only change relative to the case with low shopping costs is that now, when two or more specialists turn out to be of high value, joint profits are higher, equal to $\lambda_B u_A$ instead of $u_A - \sigma$. Since the probability of two or more specialists turning out to be of high value is increasing in k,[3] the optimal number of specialists to host is higher than in the case with low shopping costs, but still possibly below n.

Consider now the case when M can charge variable fees under hosting.

Proposition 15 *Suppose there are $n \geq 2$ specialists who offer $u_B + \Delta$ when visited outside M, but when hosted offer $u_B + \Delta$ with probability θ and u_B with probability $1-\theta$. When variable fees can be used under hosting, M prefers hosting all n specialists and hosting is jointly preferred over non-hosting if and only if*

$$\Delta > \frac{F}{(1-\lambda_A)(1-(1-\theta)^n)}.$$

Like in the benchmark case, hosting is always preferred if $F = 0$. This is not surprising: when variable fees

[3] Indeed, $1 - (1-\theta)^k - k\theta(1-\theta)^{k-1}$ is increasing in k.

can be used, whether only one or multiple hosted specialists have an efficiency advantage, M can always extract the efficiency gain σ due to hosting by setting $\tau = \sigma$.

The key difference relative to the benchmark model is that now, regardless of how many hosted specialists turn out to be of high quality, they have to compete with the outside specialists of high quality that price at cost. This explains why, going from the case with just one specialist (Proposition 5) to the case with two or more competing specialists (Proposition 15), the tradeoff shifts towards non-hosting. This holds even if $\theta = 1$, i.e. there is no uncertainty over the added value of the specialists.

The presence of the competing outside specialists constrains the price of hosted specialists. Specifically, the maximum price $\widehat{p}_S$ that can be charged by hosted specialist(s) of high quality is $\widehat{p}_S = \sigma$ if $p_A = u_A - \sigma$ (so M sells A to both types), or $\widehat{p}_S = 0$ if $p_A = u_A$ (so M sells A to B-types only). Clearly, in the latter case, M cannot extract any variable fee from the hosted specialists because they would make a loss, whereas in the former case, M can extract σ from them. Thus, it can never be profitable for M to only sell A to B-types by setting $p_A > u_A - \sigma$, so the only possible equilibrium now must involve M selling A to both types.

This has two implications. First, M's profits are $u_A - \sigma + \lambda_B \sigma$ when at least one hosted specialist turns out to be of high quality and $u_A - \sigma + \lambda_B (\sigma - \Delta)$ when none of the hosted specialists turns out to be of high quality. Thus, M will host as many specialists as possible in order to maximize the chance that at least one specialist will have an efficiency advantage. Second, the hosting vs. non-hosting tradeoff is very similar to the case $\lambda_A > \frac{\sigma}{u_A}$ in the benchmark model, i.e. the case in which M sold to A-types in equilibrium. In particular, σ has no effect on the tradeoff and the effects of the parameters Δ, λ_A and F are the same as in the benchmark model. Furthermore, taking into account the fixed cost of hosting, the tradeoff shifts towards hosting when the number of competing specialists increases, i.e. when n increases.

F.1 Proof of Propositions 14 and 15

We start with the more complicated case in which M can charge a variable fee τ when it hosts specialists, i.e. Proposition 15. We then derive the proof of Proposition 14 from the equilibrium of the subgame in which $\tau = 0$ under hosting.

Regardless of how many specialists are hosted, Bertrand competition outside the platform implies that in equilibrium we can always restrict attention to the case where all specialists (hosted and not hosted) price at zero outside the platform.

Suppose M hosts $k \in [1, n]$ specialists and has set a variable fee $\tau \geq 0$ in its contract with the hosted specialists. Given that all specialists are identical, there are only three distinct cases to consider: (i) all the hosted specialists turn out to offer u_B only, (ii) exactly one of the hosted specialists turns out to offer $u_B + \Delta$ and all other specialists offer u_B only, (iii) two or more of the hosted specialists turn out to offer $u_B + \Delta$.

Consider case (i) first. If all hosted specialists turn out to offer only u_B, then all specialists make zero profits and the outcome is as follows:

- if $\tau \geq \sigma - \Delta$, then M keeps selling to both types and makes the sales of B_M by setting $p_A = u_A - \sigma$ and $p_B = \sigma - \Delta$, obtaining profits $u_A - \sigma + \lambda_B (\sigma - \Delta)$, which is the same as without hosting.
- if $\frac{\sigma - \lambda_A u_A}{1 - \lambda_A} - \Delta \leq \tau < \sigma - \Delta$, then M keeps selling to both types and makes the sale of B_M by setting $p_A = u_A - \sigma$ and $p_B = \tau$, obtaining profits $u_A - \sigma + \lambda_B \tau$.
- if $\tau < \frac{\sigma - \lambda_A u_A}{1 - \lambda_A} - \Delta$, then M sells to B types only by setting $p_A = u_A - \tau - \Delta$ and $p_B = \tau$, obtaining profits $\lambda_B (u_A - \Delta)$.

Note that joint profits are (weakly) increasing in τ. They are maximized for $\tau \geq \sigma - \Delta$, when they are equal to $u_A - \sigma + \lambda_B (\sigma - \Delta)$.

Consider case (ii), when exactly one of the hosted specialists turns out to offer $u_B + \Delta$. With some abuse of terminology, we will refer to the specialist offering $u_B + \Delta$ as S. If $\tau > \sigma$, then S cannot make non-negative profits while avoiding that consumers prefer going to an outside specialist. The same goes for the other hosted specialists. In this case, M sets $p_A = u_A - \sigma$ and $p_B = \sigma - \Delta$ and the outcome is the same as without hosting.

Suppose then $\tau \leq \sigma$. In equilibrium, S must sell B_S to all B types because it can offer the highest utility for the B product. All other hosted specialists price at τ on M. There are two cases depending on p_A.

Suppose first that $p_A = u_A - \sigma$, so all consumers buy the A product (there is no need to set p_A any lower to attract all consumers). In this case, for S to make the sales of B_S, we must have

$$u_B + \Delta - \widehat{p}_S \geq \max\left\{u_B - p_B, u_B + \Delta - \sigma\right\}.$$

Clearly, this must hold with equality in equilibrium, otherwise S could increase $\widehat{p}_S$. Thus, we must have

$$\widehat{p}_S = \min\left\{p_B + \Delta, \sigma\right\}.$$

Furthermore, τ must not be above $\widehat{p}_S$ (so S makes non-negative profits) and M must not want to deviate by setting p_B slightly below $\widehat{p}_S - \Delta$ and selling B_M instead of getting τ from S. This means we must have

$$\min\left\{p_B, \sigma - \Delta\right\} \leq \tau \leq \min\left\{p_B + \Delta, \sigma\right\}.$$

Finally, M must not want to increase p_A and only serve B types. The best such deviation for M is to set p'_A such that

$$u_A - p'_A + u_B + \Delta - \widehat{p}_S - \sigma = u_B + \Delta - \sigma,$$

provided the solution in p'_A is below u_A. So the best deviation is

$$p'_A = u_A - \widehat{p}_S = u_A - \min\left\{p_B + \Delta, \sigma\right\}.$$

If $p_B \geq \sigma - \Delta$, then $p'_A = u_A - \sigma$, so this deviation does not do any better. If $p_B \leq \sigma - \Delta$, then $p'_A = u_A - \Delta - p_B$, so M's deviation profits are $\lambda_B\left(u_A - \Delta - p_B + \tau\right)$, whereas M's equilibrium profits are $u_A - \sigma + \lambda_B \tau$. For the deviation not to be profitable, we then need

$$\lambda_A u_A + \lambda_B\left(\Delta + p_B\right) \geq \sigma.$$

Thus, if $p_B \geq \sigma - \Delta$, then $(p_A = u_A - \sigma, \widehat{p}_S = \sigma, p_S = 0)$ is an equilibrium given τ if and only if

$$\sigma - \Delta \leq \tau \leq \sigma.$$

Equilibrium profits are $u_A - \sigma + \lambda_B \tau$ for M and $\lambda_B\left(\sigma - \tau\right)$ for S. Since these profits do not depend on p_B, we can just focus on $p_B = \sigma - \Delta$.

If $p_B \leq \sigma - \Delta$, then $(p_A = u_A - \sigma, \widehat{p}_S = \Delta + p_B, p_S = 0)$ is an equilibrium given τ if and only if

$$p_B \leq \tau \leq p_B + \Delta \text{ and } \lambda_A u_A + \lambda_B\left(\Delta + p_B\right) \geq \sigma.$$

Since joint profits are increasing in p_B, we focus on the highest possible p_B, which is $p_B = \min\left\{\tau, \sigma - \Delta\right\}$. If $\sigma - \Delta \leq \tau$, then $p_B = \sigma - \Delta$, so this is the same equilibrium as in the case $p_B \geq \sigma - \Delta$. If $\sigma - \Delta > \tau$, then

$p_B = \tau$ and $(p_A = u_A - \sigma, p_B = \tau, \widehat{p}_S = \Delta + \tau, p_S = 0)$ is an equilibrium given τ if and only if

$$\tau \geq \frac{\sigma - \lambda_A u_A}{1 - \lambda_A} - \Delta.$$

In this equilibrium, profits are $u_A - \sigma + \lambda_B \tau$ for M, $\lambda_B \Delta$ for S, and zero for all other specialists.

To conclude this case, the equilibrium with M selling to A-types exists if and only if $\tau \geq \frac{\sigma - \lambda_A u_A}{1-\lambda_A} - \Delta$ and profits are $u_A - \sigma + \lambda_B \tau$ for M and $\lambda_B \min \{\Delta, \sigma - \tau\}$ for S. Thus, joint profits are $u_A - \sigma + \lambda_B \min \{\Delta + \tau, \sigma\}$.

Suppose now $u_A - \sigma < p_A \leq u_A$, so M does not sell to A types. In this case, for S to make the sales of B_S, we must have

$$u_A - p_A + u_B + \Delta - \widehat{p}_S \geq \max \{u_A - p_A + u_B - p_B, u_B + \Delta\}.$$

Clearly, this must hold with equality in equilibrium, otherwise S could increase $\widehat{p}_S$. Thus, we must have

$$\widehat{p}_S = \min \{\Delta + p_B, u_A - p_A\}.$$

Furthermore, τ must not be above $\widehat{p}_S$ (in order that S does not make a loss) and M must not want to deviate by setting p_B slightly below $\widehat{p}_S - \Delta$ and selling B_M instead of getting τ from S. This means we must have

$$\min \{p_B, u_A - p_A - \Delta\} \leq \tau \leq \min \{\Delta + p_B, u_A - p_A\}$$

Finally, M must not want to decrease p_A to $u_A - \sigma$ and sell A to all consumers. This deviation would result in profits $u_A - \sigma + \lambda_B \tau$, whereas M's equilibrium profits are $\lambda_B (p_A + \tau)$. For this deviation not to be profitable we must have

$$u_A - p_A \leq \frac{\sigma - \lambda_A u_A}{1 - \lambda_A}.$$

There are two possibilities for this equilibrium:

- If $p_B \leq u_A - p_A - \Delta$, then this equilibrium exists if and only if

  $$p_B \leq \tau \leq p_B + \Delta \text{ and } u_A - p_A \leq \frac{\sigma - \lambda_A u_A}{1 - \lambda_A}.$$

  Equilibrium profits are then $\lambda_B (p_A + \tau)$ for M and $\lambda_B (p_B + \Delta - \tau)$ for S. Clearly, M would want to increase p_A as much as possible, so it must be that $p_A = u_A - p_B - \Delta$. Thus, for any p_B in the interval $[\tau - \Delta, \tau]$, equilibrium profits are $\lambda_B (u_A - p_B + \tau - \Delta)$ for M and $\lambda_B (p_B + \Delta - \tau)$ for S, and this is an equilibrium if and only if $p_B + \Delta \leq \frac{\sigma - \lambda_A u_A}{1-\lambda_A}$. Thus, there exists a p_B in the interval $[\tau - \Delta, \tau]$ such that this is an equilibrium if and only $\tau \leq \frac{\sigma - \lambda_A u_A}{1-\lambda_A}$. Note that joint profits are $\lambda_B u_A$ and so do not depend on p_B or τ.

- If $p_B \geq u_A - p_A - \Delta$, then this equilibrium exists if and only if

  $$u_A - p_A - \Delta \leq \tau \leq u_A - p_A \text{ and } u_A - p_A \leq \frac{\sigma - \lambda_A u_A}{1 - \lambda_A}.$$

  Equilibrium profits are then $\lambda_B (p_A + \tau)$ for M and $\lambda_B (u_A - p_A - \tau)$ for S. Clearly, M would want to increase p_A as much as possible, so it must be that $p_A = u_A - \tau$ and profits are $\lambda_B u_A$ for M and 0 for S. This is an equilibrium if and only if $\tau \leq \frac{\sigma - \lambda_A u_A}{1-\lambda_A}$.

Combining these two possibilities, we conclude that the equilibrium in which M does not sell to A types exists if and only if $\tau \leq \frac{\sigma - \lambda_A u_A}{1-\lambda_A}$ and joint profits in this equilibrium are always $\lambda_B u_A$.

Thus, summarizing case (ii), we find:

- If $\tau \leq \frac{\sigma-\lambda_A u_A}{1-\lambda_A} - \Delta$, then only the equilibrium in which M does not sell to A types exists and joint profits are $\lambda_B u_A$ (there are multiple equilibria depending on how profits are shared between M and S).
- If $\tau \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then only the equilibrium in which M sells A types exists and joint profits are $u_A - \sigma + \lambda_B \min\{\Delta+\tau, \sigma\}$.
- If $\frac{\sigma-\lambda_A u_A}{1-\lambda_A} - \Delta \leq \tau \leq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then both types of equilibria exist, so joint profits are either $\lambda_B u_A$ or $u_A - \sigma + \lambda_B \min\{\Delta+\tau, \sigma\}$.

Clearly, joint profits are weakly increasing in τ, so they are maximized for $\tau = \sigma$, which leads to profits $u_A - \lambda_A \sigma$ for M and zero for S.

Finally, consider case (iii), in which two or more of the hosted specialists turn out to offer $u_B + \Delta$. In this case, Bertrand competition pins down the prices of all the hosted specialists on M so that $\widehat{p}_S = \tau$, while Bertrand competition outside M continues to pin down the specialists' outside prices at $p_S = 0$. As in the previous case, if $\tau > \sigma$, then the hosted specialists cannot compete with the outside specialists and make non-negative profits, so the outcome is the same as under non-hosting. Suppose then $\tau \leq \sigma$. Then B-type consumers prefer buying B_S and A on M instead of only buying B_S outside if and only if $p_A + \tau \leq u_A$.

Consider first the equilibrium when M sells to A types. Then $p_A = u_A - \sigma$, so clearly $p_A + \tau \leq u_A$. In this candidate equilibrium, M makes profits $u_A - \sigma + \lambda_B \tau$. Note that in this case M does not want to deviate by setting p_B slightly below $\widehat{p}_S - \Delta = \tau - \Delta$ and selling B_M instead of getting τ from the hosted specialists. The only remaining condition for ensuring this is an equilibrium is that M does not want to increase p_A and only serve B types. The best such deviation is to set $p'_A = u_A - \tau$, leading to deviation profits $\lambda_B u_A$. This deviation is not profitable if and only if $\tau \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$.

Next consider the equilibrium when M does not sell to A types. Then $p_A = u_A - \tau \geq u_A - \sigma$. In this candidate equilibrium, M makes profits $\lambda_B u_A$. This is an equilibrium if and only if M does not want to deviate by decreasing p_A to $u_A - \sigma$ and serve both A types and B types. This condition is equivalent to $\tau \leq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$.

Thus, summarizing case (iii):

- If $\tau \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then there is a unique equilibrium in which M sells to A types by setting $p_A = u_A - \sigma$ and joint profits are $u_A - \sigma + \lambda_B \tau$
- If $\tau \leq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then there is a unique equilibrium in which M does not sell to A types involving $p_A = u_A - \tau$ and joint profits are $\lambda_B u_A$. Once again, joint profits are weakly increasing in τ and maximized for $\tau = \sigma$, when they are equal to $u_A - \lambda_A \sigma$.

Consequently, joint profits are weakly increasing in τ up to $\tau = \sigma$ in all three scenarios that can occur after uncertainty is realized. This implies that from an ex-ante perspective (before uncertainty is realized), M will set $\tau = \sigma$. Thus, when $k \geq 1$ specialists are hosted, expected joint profits are

$$\begin{aligned} E[\pi_M + \pi_S] &= (1-\theta)^k (u_A - \sigma + \lambda_B(\sigma - \Delta)) + \left(1 - (1-\theta)^k\right)(u_A - \lambda_A \sigma) \\ &= u_A - \lambda_A \sigma - (1-\theta)^k \lambda_B \Delta. \end{aligned} \tag{F.1}$$

Thus, joint profits are increasing in k, so from a joint profit perspective, M wants to host all n available specialists in order to maximize the probability of having at least one specialist of high quality on the platform. In this case, hosting is preferred to non-hosting if and only if

$$u_A - \lambda_A \sigma - (1-\theta)^n \lambda_B \Delta - F > u_A - \sigma + \lambda_B(\sigma - \Delta),$$

which is equivalent to

$$\Delta > \frac{F}{(1-\lambda_A)(1-(1-\theta)^n)}.$$

Finally, if M cannot monitor the hosted specialists' sales under hosting (Proposition 14), then the analysis above applies by imposing $\tau = 0$. The outcomes in the three cases above are now as follows:

- Case (i): none of the hosted specialists turn out to offer $u_B + \Delta$. In this case, all specialists make zero profit. If $\Delta \geq \frac{\sigma - \lambda_A u_A}{1-\lambda_A}$, then M sells A to both types and joint profits are $u_A - \sigma$. If $\Delta < \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then M sells to B types only by setting $p_A = u_A - \Delta$ and joint profits are $\lambda_B (u_A - \Delta)$. In short, joint profits are $\max\{u_A - \sigma, \lambda_B (u_A - \Delta)\}$.
- Case (ii): only one of the hosted specialists turns out to offer $u_B + \Delta$. In this case, if $\Delta \leq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, then only the equilibrium in which M sells exclusively to B types exists and joint profits are $\lambda_B u_A$. If $\sigma \leq \lambda_A u_A$, then only the equilibrium in which M sells to both A and B types exists and joint profits are $u_A - \sigma + \lambda_B \Delta$. If $\Delta \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A} \geq 0$, then both types of equilibria exist, so joint profits are either $\lambda_B u_A$ or $u_A - \sigma + \lambda_B \Delta$. And since $\Delta \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A}$, we know that $u_A - \sigma + \lambda_B \Delta \geq \lambda_B u_A$, so we assume the two firms coordinate on the equilibrium with higher joint profits, i.e. $u_A - \sigma + \lambda_B \Delta$. Thus, to summarize this case: joint profits are $\max\{\lambda_B u_A, u_A - \sigma + \lambda_B \Delta\}$.
- Case (iii): two or more hosted specialists turn out to offer $u_B + \Delta$. In this case, if $\sigma \leq \lambda_A u_A$, then joint profits are $u_A - \sigma$; if $\sigma \geq \lambda_A u_A$, then joint profits are $\lambda_B u_A$. In short, joint profits are $\max\{u_A - \sigma, \lambda_B u_A\}$.

Consequently, expected joint profits are:

$$\begin{aligned} E[\pi_M + \pi_S] &= (1-\theta)^k \max\{u_A - \sigma, \lambda_B (u_A - \Delta)\} + k(1-\theta)^{k-1}\theta \max\{\lambda_B u_A, u_A - \sigma + \lambda_B \Delta\} \\ &\quad + \left(1 - (1-\theta)^k - k\theta(1-\theta)^{k-1}\right) \max\{u_A - \sigma, \lambda_B u_A\}. \end{aligned} \tag{F.2}$$

There are therefore three cases:

- If $\frac{\sigma-\lambda_A u_A}{1-\lambda_A} \geq \Delta$, then joint profits are

$$E[\pi_M + \pi_S] = (1-\theta)^k \lambda_B (u_A - \Delta) + \left(1 - (1-\theta)^k\right) \lambda_B u_A.$$

  They are increasing in k, so joint profits are maximized by hosting all available specialists, i.e. $k^* = n$. In this case, hosting is jointly preferred to non-hosting if and only if

$$\Delta > \frac{\lambda_A (u_A - \sigma) + F}{(1-\lambda_A)(1-(1-\theta)^n)}.$$

- If $\Delta \geq \frac{\sigma-\lambda_A u_A}{1-\lambda_A} \geq 0$, then joint profits are

$$\begin{aligned} E[\pi_M + \pi_S] &= (1-\theta)^k (u_A - \sigma) + k(1-\theta)^{k-1}\theta(u_A - \sigma + \lambda_B \Delta) + \left(1 - (1-\theta)^k - k(1-\theta)^{k-1}\theta\right)\lambda_B u_A \\ &= k(1-\theta)^{k-1}\theta(\lambda_A u_A - \sigma + \lambda_B \Delta) - (1-\theta)^k (\sigma - \lambda_A u_A) + \lambda_B u_A. \end{aligned}$$

  In this case, the optimal number of specialists to host from a joint profit perspective is

$$k^* = \frac{(1-\theta)(\sigma - \lambda_A u_A)}{\theta(\lambda_A u_A - \sigma + \lambda_B \Delta)} - \frac{1}{\ln(1-\theta)}$$

and hosting is preferred to non-hosting if and only if

$$\Delta > \frac{\sigma\left(1-\lambda_A\right)-\left(1-\left(1-\theta\right)^{k^*}-k^*\left(1-\theta\right)^{k^*-1}\theta\right)\left(\sigma-\lambda_A u_A\right)+F}{\left(1+k^*\left(1-\theta\right)^{k^*-1}\theta\right)\left(1-\lambda_A\right)}$$

- If $\frac{\sigma-\lambda_A u_A}{1-\lambda_A} \leq 0$, then joint profits are

$$\begin{aligned} E\left[\pi_M+\pi_S\right] &= k\left(1-\theta\right)^{k-1}\theta\left(u_A-\sigma+\lambda_B\Delta\right)+\left(1-k\left(1-\theta\right)^{k-1}\theta\right)\left(u_A-\sigma\right) \\ &= k\left(1-\theta\right)^{k-1}\theta\lambda_B\Delta+u_A-\sigma. \end{aligned}$$

  In this case, assuming $\theta < 1$, the optimal number of specialists to host from a joint profit perspective is

$$k^* = -\frac{1}{\ln\left(1-\theta\right)}$$

  and hosting is preferred to non-hosting if and only if

$$\Delta > \frac{\sigma\left(1-\lambda_A\right)+F}{\left(1+k^*\left(1-\theta\right)^{k^*-1}\theta\right)\left(1-\lambda_A\right)}.$$

F.2 Proof of the results with multiple specialists and no uncertainty

Suppose $\theta = 1$ in the above model with multiple specialists. If both lump-sum transfers and variable fees are feasible, then (F.1) implies joint profits under hosting are

$$\pi_M+\pi_S = u_A-\lambda_A\sigma,$$

so any $k \geq 1$ yields the same joint profits.

If only lump-sum transfers are feasible, then (F.2) implies joint profits under hosting are

$$\pi_M+\pi_S = \begin{cases} \max\left\{u_A-\sigma+\lambda_B\Delta, \lambda_B u_A\right\} & \text{if} \quad k=1 \\ \\ \max\left\{u_A-\sigma, \lambda_B u_A\right\} & \text{if} \quad k>1 \end{cases}.$$

In this case, $k = 1$ is optimal under hosting, unless $\lambda_B u_A \geq u_A - \sigma + \lambda_B\Delta$, in which case any $k \geq 1$ yields the same joint profits.

Finally, if neither lump-sum transfers nor variable fees are feasible, then M's profits under hosting are

$$\pi_M = \begin{cases} \max\left\{u_A-\sigma, \lambda_B\left(u_A-\Delta\right)\right\} & \text{if} \quad k=1 \\ \\ \max\left\{u_A-\sigma, \lambda_B u_A\right\} & \text{if} \quad k>1 \end{cases}.$$

In this case, any $k > 1$ is optimal under hosting, unless $u_A - \sigma \geq \lambda_B u_A$, in which case any $k \geq 1$ yields the same joint profits.